

Soliton Solution for the Third-order AKNS Equation and its Relation with Double Wronskian Solutions

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Abstract: It is widely believed that all of the soliton solutions of the AKNS are completely described by the double Wronskian solutions. In this paper, multi-soliton solutions can also be conveniently obtained using the method of constraint. Besides each double Wronskian solution of the third-order AKNS is of invariant-manifold constraint type will be proved.

Key words: AKNS equation; double Wronskian technique; invariant-manifold constraint method

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Many physically important integrable partial differential equations belong to the AKNS hierarchy^[1-11]. The soliton solution of the AKNS hierarchy can be obtained by the inverse scattering method^[12-13]. The mathematical structures of the AKNS are still revealing^[14]. In this work we will confine ourselves to the third-order AKNS

(1)

The reason is that the soliton solutions of the third-order AKNS have been greatly simplified by the work of Yin^[15] with the double Wronskian technique^[16-19].

The Lax pair of (1) is

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_x = \begin{pmatrix} -\eta & q \\ r & \eta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_t = \begin{pmatrix} -4\eta^3 + 2qr\eta + q r_x - r q_x & 4q\eta^2 - 2q_x\eta + q_{xx} - 2q^2r \\ 4r\eta^2 + 2r_x\eta + r_{xx} - 2qr^2 & 4\eta^3 - 2qr\eta - q r_x + r q_x \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

where η is the eigenvalue parameter. n -order to obtain

the double Wronskian solution for the AKNS (1), Yin^[15] first introduced two sets of auxiliary functions ϕ and ψ

$$\phi_x = -A\phi, \psi_x = A\psi, \quad (2)$$

$$\phi_t = 4\phi_{xxx}, \psi_t = 4\psi_{xxx}, \quad (3)$$

for the third-order AKNS, where A is an arbitrary constant $(n+m+2)(n+m+2)$ matrix,

$$\phi = (\phi_1, \phi_2, \dots, \phi_{n+m+2})^T, \psi = (\psi_1, \psi_2, \dots, \psi_{n+m+2})^T,$$

$$\phi = (\phi_1, \phi_2, \dots, \phi_{n+m+2})^T, \psi = (\psi_1, \psi_2, \dots, \psi_{n+m+2})^T.$$

They proved that the third-order AKNS (1) has the following double Wronskian solution

$$q = \frac{2W^{n+2,m}(\phi; \psi)}{W^{n+1,m+1}(\phi; \psi)}, \quad r = \frac{-2W^{n,m+2}(\phi; \psi)}{W^{n+1,m+1}(\phi; \psi)}, \quad (4)$$

where $W^{j,l}(\phi; \psi)$ is defined as

$$W^{j,l}(\phi; \psi) = |\phi, \partial\phi, \dots, \partial^{j-1}\phi; \psi, \partial\psi, \dots, \partial^{l-1}\psi|, \quad (5)$$

for any non-negative integers j and l . For $l = -1$, we will define

$$|j-1; -1| = |\phi, \partial\phi, \dots, \partial^{j-1}\phi|. \quad (6)$$

Because the symbol $|j-1; -1|$ will never appear in this paper, the meaning of symbol $|j-1|$ will never be confused. Yin^[15] also showed that the double Wronskian solutions (4) include the soliton solutions,

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rational solutions, Matveev solutions and complexitons.

The constraint method has a deep theory and also a wide range of mathematical and physical significant applications^[9,20-21]. Recently Li et al^[14,22] have studied the constraints of the AKNS hierarchy. Starting from a special form of Lax pair of the AKNS hierarchy, they succeed in constraining the hierarchy to a series of univariate integrable hierarchies, which are all proven to be linearizable. The constrained AKNS hierarchy is completely solved. They first defined recursively p_k and θ_k by

$$\begin{aligned} p_{-1} &= r, p_0 = 1, p_1 = q, \\ p_{k+1} / p_k &= (qr - (\ln p_k)_{xx}) p_k / p_{k-1}, \end{aligned} \quad (7)$$

and

$$\begin{aligned} \theta_1(g) &= g, \\ \theta_{k+1}(g) &= (\ln(\theta_1(g)\theta_2(g)\cdots\theta_k(g)))_{xx} \theta_k(g). \end{aligned} \quad (8)$$

Then they proved the following theorem.

Theorem 1^[14] The constraint $\hat{L}^m q = 0$ is equivalent to $qr = (\ln p_m)_{xx}$, where $\hat{L} = (-\partial + q\partial^{-1}r)/i$, with the constraint the n -th AKNS equation is equivalent to $\partial u_m / \partial t = (-i)^{n-1} u_m^{(n)}$, where $u_m = p_{m-1} / p_m$ and p is defined by (7). Given u_m , q and r are determined by $q^{-1} = (-1)^{m-1} \theta_m(u_m)$ and $r = (-1)^m \theta_{m+1}(u_m)$ respectively, where θ is defined by (8).

We call that the solutions obtained by Theorem 1 are invariant-manifold constraint type. Of course, Theorem 1 can be applied to the third-order AKNS (1). Therefore, we have two sets of explicit solutions for the third-order AKNS (1): One set is the double Wronskian solutions (4); the other set is invariant-manifold constraint type stated in Theorem 1. Then one natural question is what the relation is between the two sets of solutions. In this work, we will prove that each double Wronskian solution of the third-order AKNS (1) is of invariant-manifold constraint type.

The paper is organized as follows. Section 2 will discuss the invariant-manifold constraint type solution of the third-order AKNS (1) and give a unified general formula for multi-soliton solutions. Section 3 will set up the relation between the double Wronskian solutions (4) and the invariant-manifold constraint type solutions.

Section 4 will give some discussions.

1 The simplest multi-soliton solutions of the third-order AKNS by constraint

According to Theorem 1, we have $q^{-1} = (-1)^{n-1} \cdot \theta_n(u_n)$ and $r = (-1)^n \theta_{n+1}(u_n)$. When $n=1$, we may take $u_1 = 1 + e^{w_1 t + k_1 x}$ and $\omega_1 = k_1^3$. Then we can obtain

$$q = \frac{1}{1 + e^{\omega_1 t + k_1 x}}, \quad r = -\frac{k_1^2 e^{k_1 x}}{e^{-\omega_1 t} + e^{k_1 x}}.$$

It is easy to check that the above solution is indeed a solution of (1), which is a one-soliton solution.

When $n=2$, we may take $u_2 = 1 + e^{w_1 t + k_1 x} + e^{w_2 t + k_2 x}$ and $\omega_i = k_i^3$, $i=1,2$. Similarly, we can get

$$\begin{aligned} q &= \frac{-(e^{-\omega_1 t - \omega_2 t} + e^{-\omega_2 t + k_1 x} + e^{-\omega_1 t + k_2 x})}{k_1^2 e^{-\omega_2 t + k_1 x} + k_2^2 e^{-\omega_1 t + k_2 x} + e^{(k_1 + k_2)x} (k_1 - k_2)^2}, \\ r &= \frac{k_1^2 k_2^2 (k_1 - k_2)^2 e^{(k_1 + k_2)x}}{k_1^2 e^{-\omega_2 t + k_1 x} + k_2^2 e^{-\omega_1 t + k_2 x} + e^{(k_1 + k_2)x} (k_1 - k_2)^2}. \end{aligned} \quad (9)$$

Also it is easy to check that the above solution is indeed a solution of (1). Solution (9) is double-soliton solution.

When $n=3$, we take $u_3 = 1 + e^{w_1 t + k_1 x} + e^{w_2 t + k_2 x} + e^{w_3 t + k_3 x}$ and $\omega_i = k_i^3$, $i=1,2,3$.

Similarly, we can get

$$\begin{aligned} q &= ((k_1 - k_2)^2 e^{(k_1 + k_2)x - \omega_3 t} + (k_2 - k_3)^2 e^{(k_2 + k_3)x - \omega_1 t}) / q^d + \\ &\quad ((k_3 - k_1)^2 e^{(k_1 + k_3)x - \omega_2 t} k_1^2 e^{-(\omega_2 + \omega_3)t + k_1 x}) / q^d + \\ &\quad (k_2^2 e^{-(\omega_1 + \omega_3)t + k_2 x} + k_3^2 e^{-(\omega_1 + \omega_2)t + k_3 x}) / q^d, \\ r &= -(k_1^2 k_2^2 k_3^2 (k_1 - k_2)^2 (k_2 - k_3)^2 \cdot \\ &\quad (k_3 - k_1)^2 e^{(k_1 + k_2 + k_3)x}) / r^d. \end{aligned} \quad (10)$$

It is easy to check that the above solution is indeed a solution of (1). Solution (10) is three-soliton solution.

We can take $u_n = 1 + e^{w_1 t + k_1 x} + e^{w_2 t + k_2 x} + \cdots + e^{w_n t + k_n x}$ and $\omega_i = k_i^3$, $i=1,2,\cdots,n$.

Although we did not give the exact proof, the above results strongly suggest that q and r have the following expressions

$$\begin{aligned} q &= \left(\sum_{i=1}^n \prod_{1 \leq j < k \leq n, j, k \neq i} (k_j - k_k)^2 e^{\sum_{i=1}^n (k_i - k_j)x - \omega_i t} \right) / q^d + \\ &\quad \sum_{i=1}^n k_i^2 e^{-(\sum_{i=1}^n \omega_i - \omega_i)t + k_i x} / q^d, \end{aligned}$$

$$r = (-1)^n (k_1^2 k_2^2 \cdots k_n^2 \prod_{1 \leq i < j \leq n} (k_i - k_j)^2 e^{\sum_{i=1}^n k_i x}) / r^d, \quad (11)$$

where

$$q^d = r^d = \prod_{1 \leq i < j \leq n} (k_i - k_j)^2 e^{\sum_{i=1}^n k_i x} + \sum_{i=1}^n \left(\prod_{1 \leq l \leq n, l \neq i} k_l^2 \prod_{1 \leq m < j \leq n, m, j \neq i} (k_m - k_j)^2 e^{\sum_{i=1}^n k_i x - \omega_i t} \right).$$

They are N -soliton solutions. The solutions (11) have been verified for $n \leq 4$. The solutions (11) are the most simplest multi-soliton solutions because $u_n = 1 + \sum_{i=1}^m e^{k_i x + \omega_i t}$, $m < n$ will result in $r=0$.

2 Relation between the two types of solutions

Theorem 2 The double Wronskian solution (4) satisfies $qr = (\ln p_{m+1})_{xx}$, where

$$p_k = 2^{k^2} \sqrt{2} \cos\left(\frac{\pi}{4} + \frac{(k-1)\pi}{2}\right) \frac{|n + \hat{k}; m - \hat{k}|}{|\hat{n}; \hat{m}|}, \quad (12)$$

and p_k satisfies (7).

Theorem 2 means that each double Wronskian solution of the third-order AKNS is of invariant-manifold constraint type.

Before proving Theorem 2, we first introduce the well-known bilinear form of (1)

$$ff_{xx} - f_x^2 + gh = 0, \quad (13)$$

$$g_t f - g f_t - g_{xxx} f + 3g_{xx} f_x - 3g_x f_{xx} + g f_{xxx} = 0, \quad (14)$$

$$h_t f - h f_t - h_{xxx} f + 3h_{xx} f_x - 3h_x f_{xx} + h f_{xxx} = 0. \quad (15)$$

The solutions of (1) can be obtained by

$$q = g / f, r = h / f. \quad (16)$$

Yin^[15] proved that the bilinear form (13)~(15) has the following double Wronskian solutions

$$f = W^{n+1, m+1}(\phi; \psi) = |n + \hat{n}; \hat{m}|, \quad (17)$$

$$g = 2W^{n+2, m}(\phi; \psi) = 2|n + \hat{n} + 1; \hat{m} - 1|, \quad (18)$$

$$h = -2W^{n, m+2}(\phi; \psi) = -2|n - \hat{n} + 1; \hat{m} + 1|. \quad (19)$$

Accordingly Equation (1) has solution (4). For convenience, we cite the following two lemmas.

Lemma 1^[12] If M is an $N \times (N-2)$ matrix and a, b, c, d are N -dimensional column vectors, then

$$|M, a, b| |M, c, d| - |M, a, c| |M, b, d| +$$

$$|M, a, d| |M, b, c| = 0. \quad (20)$$

Lemma 2^[17] Assume that is an $P = (p_{i,j})$ is $l \times l$ operator matrix and its entries $p_{i,j}$ are differential operators. $B = (b_{i,j})$ is an $l \times l$ function matrix with column vector set b_i and row vector set b_j ($i=1, 2, \dots, l; j=1, 2, \dots, l$), then

$$\sum_{i=1}^l |b_1, \dots, p_i b_i, \dots, b_l| = \sum_{i=1}^l \begin{vmatrix} b_1' \\ \vdots \\ p_i' b_i' \\ \vdots \\ b_l' \end{vmatrix}, \quad (21)$$

where

$$p_i b_i = (p_{1i} b_{1i}, p_{2i} b_{2i}, \dots, p_{li} b_{li})^T, \\ p_j' b_j' = (p_{j1}' b_{j1}', p_{j2}' b_{j2}', \dots, p_{jl}' b_{jl}')^T.$$

By Lemma 2, the following two identities can be proved in [19]

$$tr A |j; \hat{l}| = -|j - \hat{l} + 1, j + \hat{l}; \hat{k}|, \quad (22)$$

$$(tr A)^2 |j; \hat{l}| = |j - \hat{l} + 2, j, j + \hat{l}; \hat{k}| + |j - \hat{l} + 1, j + 2, \hat{k}| - 2|j - \hat{l} + 1, j + 1; \hat{k} - 1, k + 1| |j; \hat{k} - 2, k, \hat{k} + 1| + |j; \hat{k} - 1, k + 2|, \quad (23)$$

where A is a $(j + k + 2) \times (j + k + 2)$ constant matrix. If $k = -1$, then Equation (22) and Equation (23) become

$$tr A |j| = -|j - 1, j + 1|, \quad (24)$$

$$(tr A)^2 |j| = |j - 2, j, j + 1| + |j - 1, j + 2|. \quad (25)$$

In order to facilitate the writing process, we will shorten $|n + \hat{k}; m - \hat{k}| / dx$ as $|n + \hat{n}; \hat{m} - \hat{k}|_x$.

Proof of Theorem 2.

$$(1) \quad p_{-1} = r, \quad p_0 = 1, \quad \text{and} \quad p_1 = q.$$

Substituting $k = -1$ to (12), we obtain $p_{-1} = 2\sqrt{2} \cos(\pi/4 - \pi) |n - \hat{n} + 1; \hat{m} + 1| / |\hat{n}; \hat{m}| = r$. Similarly, for $k=0$, $p_0 = 1$ and for $k=1$, $p_1 = q$.

$$(2) \text{ proving } qr = (\ln p_{m+1})_{xx}.$$

By (16), $qr = (\ln p_{m+1})_{xx}$ is equivalent to

$$gh / f^2 = (\ln |n + \hat{n} + 1| / f)_{xx}. \quad (26)$$

Equation (26) is equivalent to

$$gh / f^2 + \ln(f)_{xx} = (\ln |n + \hat{n} + 1|)_{xx}. \quad (27)$$

By (13), the left side of (27) is 0. The right side of (27) can be expanded as

$$(\ln |n + \hat{n} + 1|)_{xx} = (|n + \hat{n} + 1|_{xx} |n + \hat{n} + 1|) /$$

$$|n+\hat{m}+1|^2 - |n+\hat{m}+1|_x^2 / |n+\hat{m}+1|^2, \quad (28)$$

but

$$\begin{aligned} |n+\hat{m}+1|_x &= |n+m, n+\hat{m}+2|, \\ |n+\hat{m}+1|_{xx} &= |n+\hat{m}-1, n+m+1, n+m+2| + \\ &|n+\hat{m}, n+m+3|. \end{aligned}$$

By (24) and (25) with $j=n+m+1$, (28) is 0, i.e., the right side of (27) is also 0.

$$(3) \quad p_{k+1} / p_k = (qr - (\ln p_k)_{xx}) p_k / p_{k-1}.$$

Substituting (12) to Equation (7), using (13),

Equation (7) is equivalent to

$$\begin{aligned} |n+\hat{k}; m-\hat{k}|_{xx} &= |n+\hat{k}; m-\hat{k}|-4|n+\hat{k}+1; m-\hat{k}-1| \cdot \\ &|n+\hat{k}-1; m-\hat{k}+1| = |n+\hat{k}; m-\hat{k}|_x^2. \end{aligned} \quad (29)$$

Utilizing the character of determinant we have

$$\begin{aligned} |n+\hat{k}; m-\hat{k}|_x &= |n+\hat{k}-1, n+k+1; m-\hat{k}| + \\ &|n+\hat{k}; m-\hat{k}-1, m-k+1|, \end{aligned} \quad (30)$$

and

$$\begin{aligned} |n+\hat{k}; m-\hat{k}|_{xx} &= |n+\hat{k}-2, n+k, n+k+1; m-\hat{k}| + \\ &|n+\hat{k}-1, n+k+2; m-\hat{k}| + |n+\hat{k}; m-\hat{k}-2, \\ &m-k, m-k+1| + |n+\hat{k}; m-\hat{k}-1, m-k+2| + \\ &2|n+\hat{k}-1, n+k+1; m-\hat{k}-1, m-k+1|. \end{aligned} \quad (31)$$

By Lemma 1, we have

$$\begin{aligned} |n+\hat{k}+1; m-\hat{k}-1| &= |n+\hat{k}-1; m-\hat{k}+1|, \\ |n+\hat{k}; m-\hat{k}| &= |n+\hat{k}-1, n+k+1; m-\hat{k}-1, \\ &m-k+1| - |n+\hat{k}; m-\hat{k}-1, m-k+1| \cdot \\ &|n+\hat{k}-1, n+k+1; m-\hat{k}|. \end{aligned} \quad (32)$$

Substituting (30)~(32) into (29) and simplifying, we get (29) is equivalent to

$$\begin{aligned} &(|n+\hat{k}-2, n+k, n+k+1; m-\hat{k}| + |n+\hat{k}-1, n+k+2; \\ &m-\hat{k}| + |n+\hat{k}; m-\hat{k}-2, m-k, m-k+1| + \\ &|n+\hat{k}; m-\hat{k}-1, m-k+2| - 2|n+\hat{k}-1, n+k+1; \\ &m-\hat{k}-1, m-k+1|) |n+\hat{k}; m-\hat{k}| = \\ &(|n+\hat{k}-1, n+k+1; m-\hat{k}| - \\ &|n+\hat{k}; m-\hat{k}-1, m-k+1|)^2. \end{aligned} \quad (33)$$

By (22) and (23), we know that Equation (33) is true. Therefore Equation (29) is true. So (7) must be true.

Let us give an example to demonstrate Theorem 2.

Example 1 Let A =diagonal $(0, 1, -1, 2, -2)$. Then by (2) and (3), we can sets

$$\phi = \begin{pmatrix} 1 \\ e^{-x-4t} \\ e^{x+4t} \\ e^{-2x-32t} \\ e^{2x+32t} \end{pmatrix}, \quad \psi = \begin{pmatrix} 2 \\ e^{x+4t} \\ e^{-x-4t} \\ e^{2x+32t} \\ e^{-2x-32t} \end{pmatrix}.$$

Suppose $n=1$ and $m=2$. Then by (12) we get

$$\begin{aligned} p_{-1} &= r = (6(-8 \cosh(8t+2x) + 2 \cosh(64t+4x) + 3)) / \\ &(3 \cosh(8t+2x) + \cosh(72t+6x) - 3 \cosh(64t+ \\ &4x) + 9 \cosh(56t+2x) - 10), p_0 = 1, \\ p_1 &= q = (9(\cosh(8t+2x) - \cosh(64t+4x))) / \\ &(3 \cosh(8t+2x) + \cosh(72t+6x) - 3 \cosh(64t+ \\ &4x) + 9 \cosh(56t+2x) - 10) + 1, \\ p_2 &= (48(-4 \cosh(8t+2x) + \cosh(64t+4x)) + 6)) / \\ &(3 \cosh(8t+2x) + \cosh(72t+6x) - \\ &3 \cosh(64t+4x) + 9 \cosh(56t+2x) - 10), \\ p_3 &= 18432 / (3 \cosh(8t+2x) + \cosh(72t+ \\ &6x) - 3 \cosh(64t+4x) + \\ &9 \cosh(56t+2x) - 10). \end{aligned}$$

It can be checked directly that $qr = (\ln p_3)_{xx}$ and p_{-1}, p_0, p_1, p_2 and that p_3 satisfy Equation (7).

3 Summary and discussion

In this work, we have studied two types of solutions for the third-order AKNS equation. One type is the double Wronskian solution and the other is the invariant-manifold constraint type. We have discovered the neat determinant expressions or the p_k , which are the crucial intermediate variables for the invariant-manifold constraint type solutions for the AKNS hierarchy, for the double Wronskian solutions. We have proved $qr = (\ln p_{m+1})_{xx}$ for the $(n+m+2) \times (n+m+2)$ double Wronskian solutions. This means that each double Wronskian solution for third-order AKNS equation is of invariant-manifold constraint type. Whether the two types of solutions is essentially equivalent? The question should be studied further.

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三阶 AKNS 方程的孤子解与双 Wronskian 解之间的联系

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摘要: AKNS 方程的孤子解完全可以用双朗斯基解来表示. 笔者通过约束方法便捷地求出了它的多孤子解. 同时证明了三阶 AKNS 方程的每个双朗斯基解都是不变流形的约束类型.

关键词: AKNS 方程; 双朗斯基技术; 不变流形的约束法

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