

一类广义 Camassa-Holm 型方程的尖峰孤立波和尖峰孤立波解

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摘要: 提出了包含 Camassa-Holm 方程、修正的 Camassa-Holm 方程和 Degasperis-Procesi 方程的一类非线性色散波方程, 并利用一类算子的格林函数结合方程的弱形式解, 得到了这类方程的单个尖峰孤立波解.

关键词: Camassa-Holm 方程; 修正的 Camassa-Holm 方程; Degasperis-Procesi 方程; 尖峰孤立波解
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笔者主要研究如下的非线性演化方程:

$$m_t + k((u^2 - u_x^2)m)_x + a(mu_x) + b(um_x) = 0, \\ m = u - u_{xx}, t > 0, x \in S \quad (1)$$

的尖峰孤立波解的存在性, 其中 $k \neq 0, a, b$ 为常数. 方程(1)是著名的 Camassa-Holm(CH)方程, 修正的 CH 方程和 Degasperis-Procesi(DP)方程的一个混合方程.

当 $k = 0, a = 2, b = 1$ 时, 方程(1)变成 CH 方程:

$$m_t + 2mu_x + um_x = 0, \quad m = u - u_{xx},$$

该方程是由 Camassa 和 Holm^[1]作为一种潜水波模型提出. 其实在更早时期, CH 方程被 Fokas 和 Fuchssteine 用 KdV 方程的递归算子性质得到, 他们的结果证明了 CH 方程的可积性. 关于 CH 方程的几何性质、可积性、局部和整体的存在唯一性、弱解的存在唯一性、尖峰孤子解的存在性和稳定性等已有大量的研究成果^[2-5].

当 $k = 1, a = 0, b = 0$ 时, 方程(1)变成修正的 CH 方程:

$$m_t + ((u^2 - u_x^2)m)_x = 0, \quad m = u - u_{xx},$$

该方程的可积性、对称性、适定性、尖峰孤子解的存在性及其稳定性在文献[6-8]中已有深入讨论, 结果表明修正的 CH 方程不仅具有和 CH 方程类似的性质, 还具有和 CH 方程不同的性质.

当 $k = 0, a = 3, b = 1$ 时, 方程(1)变成 DP 方程:

$$m_t + 3mu_x + um_x = 0, \quad m = u - u_{xx},$$

它是由 Degasperis 和 Procesi^[9]提出, 关于 DP 方程的几何性质、可积性、局部和整体解的存在唯一性、弱解的存在唯一性、解的爆破和波的破却、尖峰孤子解的存在性和稳定性等在文献[10-12]中已被广泛研究.

当 $a = 2b$ 时, 方程(1)变成了如下的广义 CH 型方程:

$$m_t + k((u^2 - u_x^2)m)_x + b(2mu_x + um_x) = 0, \\ m = u - u_{xx}.$$

最近这个方程的尖峰孤子解、周期尖峰孤子解及其尖峰的存在性等问题已被研究. 笔者研究了方程(1)的单个尖峰孤立波解.

1 预备知识

本文讨论方程(1)的如下初值问题:

$$m_t + k((u^2 - u_x^2)m)_x + a(mu_x) + b(um_x) = 0, \\ t > 0, x \in X, \\ u(0, x) = u_0(x), \quad m = u - u_{xx}, x \in X. \quad (2)$$

将 m 代入方程(1), 并用算子 $(1 - \partial_x^2)^{-1}$ 作用到所得方程, 有:

$$u_t + k[(u^2 - 1/3u_x^2)u_x + (1 - \partial_x^2)^{-1}(2/3u^3 + uu_x^2)_x + (1 - \partial_x^2)^{-1}(1/3u_x^3)] + b(uu_x)_x + (3b - a)(1 - \partial_x^2)^{-1}(u^2 + 1/2u_x^2)_x +$$

$$(a-2b)(1-\partial_x^2)^{-1}(3/2u^2)_x=0.$$

利用算子 $(1-\partial_x^2)^{-1}$ 的格林函数 $P(x)$ 可以表示 u 为:

$$u=(1-\partial_x^2)^{-1}m=P(x)*m,$$

其中在非周期情形, $P(x)=1/2e^{-|x|}$, 而在周期情形 $P(x)=\cosh(1/2-x)/(2\sinh(1/2))$, $*$ 表示 X 上 2 个函数的卷积, 定义为:

$$(f*g)(x)=\int_X f(y)g(x-y)dy.$$

利用上面的表述, 柯西问题(2)的弱解可如下定义.

定义 1(弱解) 给定初值 $u_0(x)\in W^{1,3}$, 函数 $u\in L^\infty([0,T],W^{1,3})$ 称为初值问题(2)的弱解, 如果对任何的函数 $\phi(t,x)\in C_c^\infty([0,T]\times X)$ 满足如下等式:

$$\begin{aligned} &\int_0^\infty\int_S[u\phi_t+k(1/3u^3\phi_t+1/3u_x^3\phi+G(x)* \\ &(2/3u^3+uu_x^2)\phi_x-G(x)*(1/3u_x^3)\phi)+ \\ &b/2u^2\phi_x+(3b-a)G(x)*(u^2+1/2u_x^2)\phi_x+ \\ &(a-2b)G(x)*(3/2u^2)\phi_x]dxdt+ \\ &\int_S u_0(0,x)\phi(0,x)dx=0. \end{aligned}$$

如果对于任何 $T>0$, u 是 $[0,T]$ 上的弱解, 则称 $u(t,x)$ 是整体弱解.

2 单个尖峰孤立波解

设方程(1)的单个尖峰孤立波解为:

$$u(t,x)=Ae^{-|x-ct|}, c\in R,$$

我们有如下定理.

定理 1 对于任意 $A\neq 0$, 方程(1)具有单个尖峰孤立波解:

$$u(t,x)=Ae^{-|x-ct|},$$

其中:

$$A=(-3b\pm\sqrt{9b^2+24kc})/4k.$$

证明

$$\text{引入 } \delta(x)=\frac{d}{dx}H(x),$$

$$\text{其中: } H(x)=\begin{cases} 0, & x<0, \\ 1, & x\geq 0, \end{cases}$$

则由 $f(x)=e^{-|x|}=e^{-H(x)x+H(-x)x}$, 得:

$$\begin{aligned} f' &= f(x)(-2\delta(x)x-H(x)+H(-x))= \\ &-\text{sign}(x)f-2\delta(x)xf, \end{aligned}$$

由 $u(t,x)=Ae^{-|x-ct|}$, 得到:

$$\begin{aligned} u_x &= A(-\text{sign}(x-ct)-2\delta(x-ct)(x-ct))e^{-|x-ct|}= \\ &-(\text{sign}(x-ct)+2\delta(x-ct)(x-ct))u, \end{aligned}$$

$$\begin{aligned} u_t &= -cA(-\text{sign}(x-ct)-2\delta(x-ct)(x-ct))e^{-|x-ct|}= \\ &c(\text{sign}(x-ct)+2\delta(x-ct)(x-ct))u, \end{aligned}$$

由定义可得 $\int_{-\infty}^{+\infty} g(x)\delta(x)dx=g(0)$.

利用分部积分法和弱解的定义(1)得出:

$$\begin{aligned} T_1 &= \int_0^\infty\int_R[u\phi_t+k(1/3u^3\phi_t+1/3u_x^3\phi)]dxdt+ \\ &\int_S u_0(0,x)\phi(0,x)dx= \\ &-\int_0^\infty\int_R[u_t+k(u^2u_x-1/3u_x^3)]\phi dxdt= \\ &-\int_0^\infty\int_R\{[c\text{sign}(x-ct)+2c(x-ct)\delta(x-ct)]u+ \\ &k[8/3(x-ct)^3\delta(x-ct)^3+4\text{sign}(x-ct)\cdot \\ &(x-ct)^2\delta(x-ct)^2-2/3\text{sign}(x-ct)]u^3\}\cdot \\ &\phi dxdt=-\int_0^\infty\int_R c\text{sign}(x-ct)u\phi dxdt- \\ &\int_0^\infty\int_R 2c(x-ct)\delta(x-ct)u\phi dxdt- \\ &\int_0^\infty\int_R 8/3k(x-ct)^3\delta(x-ct)^3u^3\phi dxdt- \\ &\int_0^\infty\int_R 4k\text{sign}(x-ct)(x-ct)^2\delta(x-ct)^2u^3\cdot \\ &\phi dxdt+\int_0^\infty\int_R 2/3k\text{sign}(x-ct)u^3\phi dxdt= \\ &-\int_0^\infty\int_R \text{sign}(x-ct)u(c-2/3ku^2)\phi dxdt, \\ T_2 &= b\int_0^\infty\int_R 1/2u^2\phi_x dxdt=-b\int_0^\infty\int_R uu_x\phi dxdt= \\ &-b\int_0^\infty\int_R (-\text{sign}(x-ct)-2(x-ct)\delta(x-ct))\cdot \\ &u^2\phi dxdt=b\int_0^\infty\int_R \text{sign}(x-ct)\phi(x)u^2 dxdt+ \\ &2b\int_0^\infty\int_R (x-ct)\delta(x-ct)\phi u^2 dxdt= \\ &b\int_0^\infty\int_R \text{sign}(x-ct)\phi(x)u^2 dxdt. \end{aligned}$$

另一方面利用分部积分法得:

$$\begin{aligned} T_3 &= \int_0^\infty\int_R k[(P(x)*(2/3u^3+uu_x^2)\phi_x- \\ &P(x)*(1/3u_x^3)\phi)]dxdt= \\ &-k\int_0^\infty\int_R [\partial_x P(x)*(2/3u^3+ \\ &uu_x^2)+P(x)*(1/3u_x^3)]\phi dxdt. \end{aligned}$$

$$\begin{aligned}
T_4 &= \int_0^\infty \int_R (3b-a)P(x) * (u^2 + 1/2u_x^2) \phi_x dx dt = \\
&\quad -(3b-a) \int_0^\infty \int_R \partial_x P(x) * (u^2 + 1/2u_x^2) \phi dx dt, \\
T_5 &= \int_0^\infty \int_R (a-2b)P(x) * (3/2u^2) \phi_x dx dt = \\
&\quad -(a-2b) \int_0^\infty \int_R \partial_x P(x) * (3/2u^2) \phi dx dt.
\end{aligned}$$

令

$$\begin{aligned}
R_1 &= \partial_x P(x) * (2/3u^3 + uu_x^2) + P(x) * (1/3u_x^3), \\
R_2 &= \partial_x P(x) * (u^2 + 1/2u_x^2), \\
R_3 &= \partial_x P(x) * (3/2u^2).
\end{aligned}$$

注意到格林函数 $P(x)$ 的导数为:

$$\begin{aligned}
\partial_x P(x) &= -1/2(\text{sign}(x) + 2\delta(x)x)e^{-|x|} = \\
&\quad -1/2\text{sign}(x)e^{-|x|} - \delta(x)xe^{-|x|}, x \in R,
\end{aligned}$$

则

$$\begin{aligned}
R_1 &= \partial_x P(x) * (2/3u^3 + uu_x^2) + P(x) * (1/3u_x^3) = \\
&\quad (-1/2\text{sign}(x)e^{-|x|} - \delta(x)xe^{-|x|}) * \\
&\quad (2/3u^3 + uu_x^2) + 1/2e^{-|x|} * (1/3u_x^3) = \\
&\quad -1/2\text{sign}(x)e^{-|x|} * (2/3u^3 + uu_x^2) - \\
&\quad \delta(x)xe^{-|x|} * (2/3u^3 + uu_x^2) + \\
&\quad 1/2e^{-|x|} * (1/3u_x^3) = R_{11} + R_{12} + R_{13},
\end{aligned}$$

$$\begin{aligned}
R_2 &= \partial_x P(x) * (u^2 + 1/2u_x^2) = \\
&\quad (-1/2\text{sign}(x)e^{-|x|} - \delta(x)xe^{-|x|}) * \\
&\quad (u^2 + 1/2u_x^2) = -1/2\text{sign}(x)e^{-|x|} * \\
&\quad (u^2 + 1/2u_x^2) - \delta(x)xe^{-|x|} * \\
&\quad (u^2 + 1/2u_x^2) = R_{21} + R_{22}, \\
R_3 &= \partial_x P(x) * (3/2u^2) = \\
&\quad (-1/2\text{sign}(x)e^{-|x|} - \delta(x)xe^{-|x|}) * (3/2u^2) = \\
&\quad -1/2\text{sign}(x)e^{-|x|} * (3/2u^2) - \delta(x)xe^{-|x|} * \\
&\quad (3/2u^2) = R_{31} + R_{32}.
\end{aligned}$$

首先可以求出

$$\begin{aligned}
R_{12} &= -\delta(x)xe^{-|x|} * (2/3u^3 + uu_x^2) = \\
&\quad -\int_{-\infty}^{+\infty} \delta(x-y)(x-y)e^{-|x-y|} \cdot \\
&\quad (2/3u^3 + uu_x^2)|_{x=y} dy = 0. \\
R_{22} &= -\delta(x)xe^{-|x|} * (u^2 + 1/2u_x^2) = \\
&\quad -\int_{-\infty}^{+\infty} \delta(x-y)(x-y)e^{-|x-y|} \cdot
\end{aligned}$$

$$(u^2 + 1/2u_x^2)|_{x=y} dy = 0,$$

$$\begin{aligned}
R_{32} &= -\delta(x)xe^{-|x|} * (3/2u^2) = \\
&\quad -\int_{-\infty}^{+\infty} \delta(x-y)(x-y)e^{-|x-y|} \cdot \\
&\quad (3/2u^2)|_{x=y} dy = 0.
\end{aligned}$$

当 $x > ct$ 时,

$$\begin{aligned}
R_{11} &= -1/2\text{sign}(x)e^{-|x|} * (2/3u^3 + uu_x^2) = \\
&\quad -1/2 \int_{-\infty}^{+\infty} \text{sign}(x-y)e^{-|x-y|} \cdot \\
&\quad (4(y-ct)^2 \delta(y-ct)^2 + \\
&\quad 4\text{sign}(y-ct)(y-ct) \cdot \\
&\quad \delta(y-ct) + 5/3) A^3 e^{-3|y-ct|} dy = -5/6 \cdot \\
&\quad \int_{-\infty}^{+\infty} \text{sign}(x-y)e^{-|x-y|} A^3 e^{-3|y-ct|} dy - \\
&\quad 2 \int_{-\infty}^{+\infty} \text{sign}(x-y)e^{-|x-y|} \delta(y-ct)^2 \cdot \\
&\quad (y-ct)^2 A^3 e^{-3|y-ct|} dy - \\
&\quad 2 \int_{-\infty}^{+\infty} \text{sign}(x-y)e^{-|x-y|} (y-ct) \cdot \\
&\quad \delta(y-ct) A^3 e^{-3|y-ct|} dy = -5/6 \cdot \\
&\quad \int_{-\infty}^{+\infty} \text{sign}(x-y)e^{-|x-y|} A^3 e^{-3|y-ct|} dy = \\
&\quad -5/6 A^3 (\int_{-\infty}^{ct} + \int_{ct}^x + \int_x^{+\infty}) \text{sign}(x-y) \cdot \\
&\quad e^{-|x-y|} e^{-3|y-ct|} dy = J_1 + J_2 + J_3.
\end{aligned}$$

分别计算 J_1, J_2, J_3 , 有:

$$\begin{aligned}
J_1 &= -5/6 A^3 \int_{-\infty}^{ct} e^{-(x-y)} e^{-3(ct-y)} dy = \\
&\quad -5/6 A^3 e^{-x-3ct} \int_{-\infty}^{ct} e^{4y} dy = -5/24 A^3 e^{-x+ct},
\end{aligned}$$

$$\begin{aligned}
J_2 &= -5/6 A^3 \int_{ct}^x e^{-(x-y)} e^{-3(y-ct)} dy = \\
&\quad 5/12 A^3 e^{-3x+3ct} - 5/12 A^3 e^{-x+ct},
\end{aligned}$$

和

$$\begin{aligned}
J_3 &= 5/6 A^3 \int_x^{+\infty} e^{-(y-x)} e^{-3(y-ct)} dy = \\
&\quad 5/6 A^3 e^{x+3ct} \int_x^{+\infty} e^{-4y} dy = 5/24 A^3 e^{-3x+3ct}.
\end{aligned}$$

把 J_1, J_2, J_3 代入 R_{11} , 可得:

$$R_{11} = 5/8 A^3 e^{-3x+3ct} - 5/8 A^3 e^{-x+ct}. \quad (3)$$

用同样的方法, 计算可得:

$$\begin{aligned}
R_{13} &= 1/2 e^{-|x|} * (1/3u_x^3) = \\
&\quad 1/2 \int_{-\infty}^{+\infty} e^{-|x-y|} (-8/3(y-ct)^3 \cdot
\end{aligned}$$

$$\begin{aligned}
& \delta(y-ct)^3 - 4\text{sign}(y-ct) \cdot \\
& (y-ct)^2 \delta(y-ct)^2 - 2(y-ct) \cdot \\
& \delta(y-ct) - 1/3\text{sign}(y-ct) \cdot \\
& A^3 e^{-3|y-ct|} dy = 1/2[-8/3 \cdot \\
& \int_{-\infty}^{+\infty} e^{-|x-y|} (y-ct)^3 \delta(y-ct)^3 \cdot \\
& A^3 e^{-3|y-ct|} dy - 4 \int_{-\infty}^{+\infty} \text{sign}(y-ct) \cdot \\
& e^{-|x-y|} \delta(y-ct)^2 (y-ct)^2 A^3 e^{-3|y-ct|} dy - \\
& 2 \int_{-\infty}^{+\infty} e^{-|x-y|} (y-ct) \delta(y-ct) A^3 e^{-3|y-ct|} dy - \\
& 1/3 \int_{-\infty}^{+\infty} e^{-|x-y|} \text{sign}(y-ct) A^3 e^{-3|y-ct|} dy = \\
& -1/6 \int_{-\infty}^{+\infty} e^{-|x-y|} \text{sign}(y-ct) A^3 e^{-3|y-ct|} dy = \\
& -1/6 A^3 \left(\int_{-\infty}^{ct} + \int_{ct}^x + \int_x^{+\infty} \right) \text{sign}(y-ct) \cdot \\
& e^{-|x-y|} e^{-3|y-ct|} dy = \\
& J_4 + J_5 + J_6.
\end{aligned}$$

其中:

$$\begin{aligned}
J_4 &= 1/6 A^3 \int_{-\infty}^{ct} e^{-(x-y)} e^{-3(ct-y)} dy = \\
& 1/3 A^3 e^{-x-3ct} \int_{-\infty}^{ct} e^{4y} dy = \\
& 1/24 A^3 e^{-x+ct}, \\
J_5 &= -1/6 A^3 \int_{ct}^x e^{-(x-y)} e^{-3(y-ct)} dy = \\
& 1/12 A^3 e^{-3x+3ct} - 1/12 A^3 e^{-x+ct}, \\
J_6 &= -1/6 A^3 \int_x^{+\infty} e^{-(y-x)} e^{-3(y-ct)} dy = \\
& -1/6 A^3 e^{x+3ct} \int_x^{+\infty} e^{-4y} dy = \\
& -1/24 A^3 e^{-3x+3ct},
\end{aligned}$$

则

$$\begin{aligned}
R_{13} &= J_4 + J_5 + J_6 = \\
& 1/24 A^3 e^{-3x+3ct} - 1/24 A^3 e^{-x+ct},
\end{aligned}$$

代入计算, 得:

$$\begin{aligned}
R_1 &= R_{11} + R_{12} + R_{13} = \\
& 2/3 A^3 e^{-3x+3ct} - 2/3 A^3 e^{-x+ct}.
\end{aligned}$$

其次可以求出

$$\begin{aligned}
R_{21} &= -1/2 \text{sign}(x) e^{-|x|} * (u^2 + 1/2 u_x^2) = \\
& -1/2 \text{sign}(x) e^{-|x|} * (3/2 + 2\delta(x-ct)^2 \cdot \\
& (x-ct)^2 + 2\text{sign}(x-ct)\delta(x-ct) \cdot
\end{aligned}$$

$$\begin{aligned}
& (x-ct)u^2 = -1/2 \text{sign}(x) e^{-|x|} * \\
& (3/2 u^2) - \text{sign}(x) e^{-|x|} * \\
& [\delta(x-ct)^2 (x-ct)^2 u^2] - \text{sign}(x) e^{-|x|} * \\
& [\text{sign}(x-ct)\delta(x-ct)(x-ct)u^2] = \\
& -1/2 \text{sign}(x) e^{-|x|} * (3/2 u^2) - \\
& \int_{-\infty}^{+\infty} \text{sign}(x-y) e^{-|x-y|} \cdot \\
& \delta(y-ct)^2 (y-ct)^2 u^2 dy - \\
& \int_{-\infty}^{+\infty} \text{sign}(x-y) e^{-|x-y|} \cdot \\
& \text{sign}(y-ct)\delta(y-ct)(y-ct)u^2 dy = \\
& -1/2 \text{sign}(x) e^{-|x|} * (3/2 u^2).
\end{aligned}$$

则

$$\begin{aligned}
R_2 &= R_3 = \\
& -1/2 \text{sign}(x) e^{-|x|} * (3/2 u^2) = \\
& -A^2 e^{-x+ct} + A^2 e^{-2x+2ct}. \quad (4)
\end{aligned}$$

同样, 当 $x < ct$ 时, 可得:

$$R_1 = -2/3 A^3 e^{3x-3ct} + 2/3 A^3 e^{x-ct}, \quad (5)$$

$$R_2 = R_3 = A^2 e^{x-ct} - A^2 e^{2x-2ct}, \quad (6)$$

因此把(3)~(6)式代入 T_3, T_4, T_5 , 得到:

$$\begin{aligned}
T_3 &= -k \int_0^\infty \int_R [\partial_x P(x) * (2/3 u^3 + u u_x^2) + \\
& P(x) * (1/3 u_x^3)] \phi dx dt = \\
& -2/3 k \int_0^\infty \int_R \text{sign}(x-ct)(u^3 - A^2 u) \phi dx dt, \quad (7)
\end{aligned}$$

$$\begin{aligned}
T_4 + T_5 &= -b \int_0^\infty \int_R \partial_x P(x) * (3/2 u^2) \phi dx dt = \\
& -b \int_0^\infty \int_R \text{sign}(x-ct)(u^2 - A u) \phi dx dt. \quad (8)
\end{aligned}$$

最后把 T_1 和 T_2 , (7)式和(8)式代入定义 1 方程, 得到:

$$\begin{aligned}
& -(c - 2/3 k u^2) - 2/3 k (u^2 - A^2) + \\
& b u - b(u - A) = \\
& -c + 2/3 k A^2 + b A = 0,
\end{aligned}$$

解

$$2kA^2 + 3bA - 3c = 0,$$

得

$$A = \frac{-3b \pm \sqrt{9b^2 + 24kc}}{4k}.$$

定理得证.

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Peaked and Periodic Peaked Traveling Wave Solutions for a Generalized Camassa-Holm Type Equation

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Abstract: In this paper, the nonlinear dispersive equations with type-I, including the Camassa-Holm equation, the modified Camassa-Holm equation and the Degasperis-Procesi equation, are proposed. By using the Green function with type-I operator and combining the weak solution of the equation, the peaked traveling wave solution is obtained for the given nonlinear dispersive equations.

Key words: Camassa-Holm equation; modified Camassa-Holm equation; Degasperis-Procesi equation; peaked traveling wave solution

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