

二元均匀分布的识别性

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摘要: 本文分别讨论了单参数二元均匀分布的识别性、双参数二元均匀分布的识别性、叁参数二元均匀分布的识别性, 对于单参数二元均匀分布, 已知最小值的分布时, 则参数可识别; 对于双参数二元均匀分布, 已知可识最小值的分布密度时, 则参数可识别; 对于叁参数二元均匀分布, 已知可识最小值的分布密度时, 所有参数皆不可识别.

关键词: 二元均匀分布; 识别性; 最小值随机变量

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自从 1978 年, 文献[1]完整地解决了二元正态分布的参数的识别性问题以来, 有关分布参数识别性的文献已有不少^[2-10], 这些文献讨论的是三元及多元正态分布的识别性, 涉及二元均匀分布识别性的相关文献至今未见. 众所周知, 均匀分布与正态分布同为概率统计教学中二个最为常用的分布, 有标准正态分布函数值表, 有区间 $[0, 1]$ 上的均匀分布的随机数表, 文献[11]把均匀分布与正态分布放在一起进行教学设计. 在统计研究领域, 从 1980 年以来, 文献[12]讨论了若干个二元均匀分布的性质. 文献[13]基于一元均匀分布的特征研究了二元均匀分布的特征. 文献[14]提出了一类对称二元均匀分布, 并讨论了这个对称二元均匀分布的相关结构. 文献[15]说明了在二元指数分布情形下, 二个随机变量之 Kendall 相关系数 τ 恰好是通常意义下的相关系数. 文献[16]表明: 二元指数分布的参数估计可以通过识别性途径得到完美地解决. 而文献[17]不能够获得二元指数分布参数的最大似然估计. 由于参数的可识别直接导致参数的可估计, 而参数估计是模型求解的核心问题之一, 因此研究二元均匀分布的识别性具有一定的必要性. 在本文中, 分别讨论了单参数二元均匀分布的识别性、双参数二元均匀分布的识别性、叁参数二元均匀分布的识别性.

1 单参数二元均匀分布的识别性

定义 1 称 (X_1, X_2) 服从以原点为中心的圆上的二元均匀分布, 其联合密度函数为:

$$f(x, y) = \begin{cases} \frac{1}{\pi a^2}, & x^2 + y^2 \leq a^2 \\ 0, & \text{其他} \end{cases}$$

记作 $(X_1, X_2) \sim BUD_C(a)$.

引理 1 若 $(X_1, X_2) \sim BUD_C(a)$, 则 Z 的分布密度为:

$$g(z) = 2\left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2}\right), |z| \leq a.$$

证明 以原点为中心的圆上的二元均匀分布,

$$f(x, y) = \begin{cases} \frac{1}{\pi a^2}, & x^2 + y^2 \leq a^2 \\ 0, & \text{其他} \end{cases}$$

$$\bar{G}(z) = P(X > z, Y > z) = \int_z^{\sqrt{a^2 - z^2}} dx \int_z^{\sqrt{a^2 - x^2}} \frac{dy}{\pi a^2} =$$

$$\int_z^{\sqrt{a^2 - z^2}} \frac{\sqrt{a^2 - x^2} - z}{\pi a^2} dx, 0 \leq z \leq a;$$

$$\bar{G}(z) = \int_z^a dx \int_0^{\sqrt{a^2 - x^2}} \frac{1}{\pi a^2} dy + \int_z^0 dy \int_z^{\sqrt{a^2 - y^2}} \frac{1}{\pi a^2} dx =$$
$$\int_z^a \frac{\sqrt{a^2 - x^2}}{\pi a^2} dx + \int_z^0 \frac{\sqrt{a^2 - y^2} - z}{\pi a^2} dy, -a \leq z < 0.$$

Z 的分布密度为:

$$g(z) = \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right) + \int_z^{\sqrt{a^2 - z^2}} \frac{1}{\pi a^2} dx =$$

$$2 \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right), 0 \leq z \leq a,$$

$$g(z) = \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right) + \frac{\sqrt{a^2 - z^2}}{\pi a^2} + \int_z^0 \frac{1}{\pi a^2} dy =$$

$$2 \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right), -a \leq z < 0,$$

$$\text{合并得: } g(z) = 2 \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right), |z| \leq a.$$

定理 1 若 $(X_1, X_2) \sim BUD_C(a)$, $(X'_1, X'_2) \sim BUD_C(a')$, 已知 Z 与 Z' 同分布, 则参数 a 可识别.

证明 由 $g(z) = g'(z)$, 得

$$2 \left(\frac{\sqrt{a^2 - z^2} - z}{\pi a^2} \right) = 2 \left(\frac{\sqrt{a'^2 - z^2} - z}{\pi a'^2} \right),$$

得 $a = a'$.

定义 2 称 (X_1, X_2) 服从以原点为顶点; 一边在 X 轴上, 一边在 Y 轴上的正方形区域上的二元均匀分布, 其联合密度函数为:

$$f(x, y) = \begin{cases} \frac{1}{a^2}, & 0 \leq x \leq a, 0 \leq y \leq a, \\ 0, & \text{其他} \end{cases}$$

记作 $(X_1, X_2) \sim BUD_S(a)$.

引理 2 若 $(X_1, X_2) \sim BUD_S(a)$, 则 Z 的分布密度为: $g(z) = 2(1 - \frac{z}{a}) \frac{1}{a} = \frac{2(a - z)}{a^2}, 0 \leq z \leq a$.

证明

$$f(x, y) = \begin{cases} \frac{1}{a^2}, & 0 \leq x \leq a, 0 \leq y \leq a, \\ 0, & \text{其他} \end{cases}$$

Z 的尾概率为:

$$\bar{G}(z) = \int_z^a dx \int_z^a \frac{1}{a^2} dy = \int_z^a \frac{a - z}{a^2} dx = (1 - \frac{z}{a})^2;$$

Z 的分布密度为:

$$g(z) = 2(1 - \frac{z}{a}) \frac{1}{a} = \frac{2(a - z)}{a^2}, 0 \leq z \leq a.$$

定理 2 若 $(X_1, X_2) \sim BUD_S(a)$, $(X'_1, X'_2) \sim BUD_S(a')$, 已知 Z 与 Z' 同分布, 则参数 a 可识别.

证明 由 $g(z) = g'(z)$, 得

$$\frac{2(a - z)}{a^2} = \frac{2(a' - z)}{a'^2},$$

得 $a = a'$.

2 双参数二元均匀分布的识别性

定义 3 称 (X_1, X_2) 服从以原点为中心的椭圆上的二元均匀分布, 其联合密度函数为:

$$f(x, y) = \begin{cases} \frac{1}{\pi ab}, & \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, \\ 0, & \text{其他} \end{cases}$$

记作 $(X_1, X_2) \sim BUD_E(a, b)$.

引理 3 若 $(X_1, X_2) \sim BUD_E(a, b)$, 则 Z 的分布密度为:

$$g(z) = \left(\frac{a\sqrt{a^2 - z^2} - bz}{\pi a^2 b} \right) + \left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right),$$

$$-\max(a, b) \leq z \leq \min(a, b).$$

证明 以原点为中心的椭圆上二元均匀分布,

$$f(x, y) = \begin{cases} \frac{1}{\pi ab}, & \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, \\ 0, & \text{其他} \end{cases}$$

$$\bar{G}(z) = P(X > z, Y > z) = \int_z^a \sqrt{1 - \frac{z^2}{b^2}} dx \int_z^b \sqrt{1 - \frac{x^2}{a^2}} \frac{1}{\pi ab} dy =$$

$$\int_z^a \sqrt{1 - \frac{z^2}{b^2}} \frac{b\sqrt{1 - x^2/a^2} - z}{\pi ab} dx, 0 \leq z < \min(a, b),$$

$$\bar{G}(z) = \int_z^a dx \int_0^b \sqrt{1 - \frac{x^2}{a^2}} \frac{1}{\pi ab} dy + \int_z^0 dy \int_z^a \sqrt{1 - \frac{y^2}{b^2}} \frac{1}{\pi ab} dx =$$

$$\int_z^a \frac{(b/a)\sqrt{a^2 - x^2}}{\pi ab} dx + \int_z^0 \frac{a\sqrt{1 - y^2/b^2} - z}{\pi ab} dy,$$

$$-\max(a, b) \leq z < 0.$$

Z 的分布密度为:

$$g(z) = \left(\frac{b\sqrt{1 - z^2/a^2} - z}{\pi ab} \right) + \int_z^a \sqrt{1 - \frac{z^2}{b^2}} \frac{1}{\pi ab} dx =$$

$$\left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right) + \left(\frac{a\sqrt{b^2 - z^2} - bz}{\pi ab^2} \right),$$

$$0 \leq z \leq \min(a, b),$$

$$g(z) = \left(\frac{b\sqrt{a^2 - z^2}}{\pi a^2 b} \right) + \left(\frac{a\sqrt{a^2 - z^2} - bz}{\pi ab^2} \right) +$$

$$\int_z^0 \frac{1}{\pi ab} dy = \left(\frac{a\sqrt{a^2 - z^2} - bz}{\pi a^2 b} \right) +$$

$$\left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right), -\max(a, b) \leq z < 0,$$

$$\text{合并得 } g(z) = \left(\frac{a\sqrt{a^2 - z^2} - bz}{\pi a^2 b} \right) + \left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right),$$

$$-\max(a, b) \leq z \leq \min(a, b).$$

定理 3 若 $(X_1, X_2) \sim BUD_E(a, b)$, $(X'_1, X'_2) \sim BUD_E(a', b')$, 若已知 Z 与 Z' 同分布, 则参数 a, b 皆不可识别.

引理 4 若 $(X_1, X_2) \sim BUD_E(a, b)$, 则 (Z, I) 的分布密度为:

$$p_i f_i(z) = \begin{cases} \left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right), i=1, \\ -\max(a, b) \leq z \leq \min(a, b) \\ \left(\frac{a\sqrt{b^2 - z^2} - bz}{\pi ab^2} \right), i=2, \\ -\max(a, b) \leq z \leq \min(a, b) \end{cases}.$$

证明

$$P(Z > z, I = 1) = P(Y > X, X > z) =$$

$$\int_z^{\frac{ab}{\sqrt{a^2+b^2}}} dx \int_x^b \sqrt{1 - \frac{x^2}{a^2}} \frac{1}{\pi ab} dy =$$

$$\int_z^{\frac{ab}{\sqrt{a^2+b^2}}} \frac{1}{\pi ab} (b\sqrt{1 - \frac{x^2}{a^2}} - x) dx,$$

$$-\max(a, b) \leq z \leq \min(a, b);$$

$$P(Z > z, I = 2) = P(X > Y, Y > z) =$$

$$\int_z^{\frac{ab}{\sqrt{a^2+b^2}}} dy \int_y^a \sqrt{1 - \frac{y^2}{b^2}} \frac{1}{\pi ab} dx =$$

$$\int_z^{\frac{ab}{\sqrt{a^2+b^2}}} \frac{1}{\pi ab} (a\sqrt{1 - \frac{y^2}{b^2}} - y) dy,$$

$$-\max(a, b) \leq z \leq \min(a, b),$$

关于 z 求导, 合并得:

$$p_i f_i(z) = \begin{cases} \left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right), i=1, \\ -\max(a, b) \leq z \leq \min(a, b) \\ \left(\frac{a\sqrt{b^2 - z^2} - bz}{\pi ab^2} \right), i=2, \\ -\max(a, b) \leq z \leq \min(a, b) \end{cases}.$$

定理 4 若 $(X_1, X_2) \sim BUD_E(a, b)$, $(X'_1, X'_2) \sim BUD_E(a', b')$, 若已知 (Z, I) 与 (Z', I') 同分布, 则所有参数皆可识别.

证明 由 $p_i f_i(z) = p'_i f'_i(z)$, 得

$$\begin{cases} \left(\frac{b\sqrt{a^2 - z^2} - az}{\pi a^2 b} \right) = \frac{b'\sqrt{a'^2 - z^2} - a'z}{\pi a'^2 b'} \\ \left(\frac{a\sqrt{b^2 - z^2} - bz}{\pi b^2 a} \right) = \frac{a'\sqrt{b'^2 - z^2} - b'z}{\pi b'^2 a'} \end{cases},$$

得

$$\begin{cases} \frac{1}{\pi ab} = \frac{1}{\pi a' b'} \\ \frac{1}{\pi b^2} = \frac{1}{\pi b'^2} \end{cases},$$

得

$$\begin{cases} a = a' \\ b = b' \end{cases}.$$

定义 4 称 (X_1, X_2) 服从以原点为顶点, 一边在 X 轴, 一边在 Y 轴的长方形上的二元均匀分布, 其密度函数为:

$$f(x, y) = \begin{cases} \frac{1}{ab}, 0 \leq x \leq a, 0 \leq y \leq b \\ 0, \text{其他} \end{cases},$$

记作 $(X_1, X_2) \sim BUD_R(a, b)$.

引理 5 若 $(X_1, X_2) \sim BUD_R(a, b)$, 则 Z 的分布密度为: $g(z) = \frac{1}{a} + \frac{1}{b} - \frac{2z}{ab}, 0 \leq z \leq \min(a, b)$.

证明 Z 的尾概率为:

$$\begin{aligned} \bar{G}(z) &= \int_z^a dx \int_z^b \frac{1}{ab} dy = \int_z^a \left(\frac{1}{a} - \frac{z}{ab} \right) dx = \\ &1 - \frac{z}{a} - \frac{z}{b} + \frac{z^2}{ab}; \end{aligned}$$

Z 的分布密度为:

$$g(z) = \frac{1}{a} + \frac{1}{b} - \frac{2z}{ab}, 0 \leq z \leq \min(a, b).$$

定理 5 若 $(X_1, X_2) \sim BUD_R(a, b)$, $(X'_1, X'_2) \sim BUD_R(a', b')$, 若已知 Z 与 Z' 同分布, 则参数 a, b 皆不可识别.

引理 6 若 $(X_1, X_2) \sim BUD_R(a, b)$, 则 (Z, I) 的分布密度为:

$$p_i f_i(z) = \begin{cases} \frac{1}{a} - \frac{z}{ab}, i=1, 0 \leq z \leq \min(a, b) \\ \frac{1}{b} - \frac{z}{ab}, i=2, 0 \leq z \leq \min(a, b) \end{cases}.$$

证明

$$P(Z > z, I = 1) = P(Y > X, X > z) =$$

$$\int_z^b dy \int_z^y \frac{1}{ab} dx = \frac{b^2 - z^2}{2ab} - \frac{z}{a} + \frac{z^2}{ab},$$

同理:

$$P(Z > z, I = 2) = P(X > Y, Y > z) =$$

$$\int_z^b dy \int_y^a \frac{1}{ab} dx = 1 - \frac{z}{b} - \frac{b^2}{2ab} + \frac{z^2}{2ab},$$

关于 z 求导, 合并得:

$$p_i f_i(z) = \begin{cases} \frac{1}{a} - \frac{z}{ab}, i=1, 0 \leq z \leq \min(a, b) \\ \frac{1}{b} - \frac{z}{ab}, i=2, 0 \leq z \leq \min(a, b) \end{cases}.$$

定理 6 若 $(X_1, X_2) \sim BUD_R(a, b), (X'_1, X'_2) \sim BUD_R(a', b')$, 若已知 (Z, I) 与 (Z', I') 同分布, 则所有参数皆可识别.

证明 由 $p_i f_i(z) = p'_i f'_i(z)$, 得

$$\begin{cases} \frac{1}{a} - \frac{z}{ab} = \frac{1}{a'} - \frac{z}{a'b'} \\ \frac{1}{b} - \frac{z}{ab} = \frac{1}{b'} - \frac{z}{a'b'} \end{cases},$$

得

$$\begin{cases} a = a' \\ b = b' \end{cases}.$$

定义 5 称 (X_1, X_2) 服从以原点为顶点, 一边在 X 轴上, 顶点为 $P(a, 0)$, 一边在 Y 轴上, 顶点为 $Q(0, b)$ 的直角三角形 G_1 上的二元均匀分布, 其联合密度函数为:

$$f(x, y) = \begin{cases} \frac{2}{ab}, (x, y) \in G_1 \\ 0, \text{其他} \end{cases},$$

记作 $(X_1, X_2) \sim BUD_T(a, b)$.

引理 7 若 $(X_1, X_2) \sim BUD_T(a, b)$, 则 Z 的分布密度为:

$$g(z) = \frac{2}{a} + \frac{2}{b} - \left(\frac{2}{a^2} + \frac{4}{ab} + \frac{2}{b^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}.$$

证明 Z 的尾概率为:

$$\begin{aligned} \bar{G}(z) &= \int_z^{a(1-\frac{z}{b})} dx \int_z^{\frac{b}{a}x+b} \frac{2}{ab} dy = \int_z^{a(1-\frac{z}{b})} \left(\frac{2}{a} - \frac{2x}{a^2} - \frac{2z}{ab} \right) dx \\ &= \left(\frac{2}{a} - \frac{2z}{ab} \right) \left(a - \frac{az}{b} - z \right) - \frac{1}{a^2} \left(\left(a - \frac{az}{b} \right)^2 - z^2 \right), 0 \leq z \leq \frac{ab}{a+b}, \end{aligned}$$

Z 的分布密度为:

$$\begin{aligned} g(z) &= - \left(- \frac{2}{ab} \left(a - \frac{az}{b} - z \right) - \left(\frac{a}{b} + 1 \right) \left(\frac{2}{a} - \frac{2z}{ab} \right) - \frac{1}{a^2} \left(2 \left(a - \frac{az}{b} \right) \times \left(- \frac{a}{b} \right) - 2z \right) \right) \\ &= \frac{2}{a} + \frac{2}{b} - \left(\frac{2}{a^2} + \frac{4}{ab} + \frac{2}{b^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}. \end{aligned}$$

定理 7 若 $(X_1, X_2) \sim BUD_T(a, b), (X'_1, X'_2) \sim BUD_T(a', b')$, 若已知 Z 与 Z' 同分布, 则参数 a, b

皆不可识别.

引理 8 若 $(X_1, X_2) \sim BUD_T(a, b)$, 则 (Z, I) 的分布密度为:

$$p_i f_i(z) = \begin{cases} \frac{2}{a} - \left(\frac{2}{ab} + \frac{2}{a^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}, i=1 \\ \frac{2}{b} - \left(\frac{2}{ab} + \frac{2}{b^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}, i=2 \end{cases}.$$

证明 下面求 (Z, I) 的联合密度:

$$P(Z > z, I = 1) = P(Y > X, X > z) = \int_z^{\frac{ab}{a+b}} dx.$$

$$\begin{aligned} \int_x^{\frac{b}{a}x+b} \frac{2}{ab} dy &= \int_z^{\frac{ab}{a+b}} \left(\frac{2}{a} - \frac{2(a+b)x}{a^2b} \right) dx = \\ &= \frac{2}{a} \left(\frac{ab}{a+b} - z \right) - \frac{a+b}{a^2b} \left(\left(\frac{ab}{a+b} \right)^2 - z^2 \right), \\ &0 \leq z \leq ab / (a+b), \end{aligned}$$

$$\begin{aligned} p_1 f_1(z) &= \frac{2}{a} - \frac{2(a+b)z}{a^2b} = \frac{2}{a} - \left(\frac{2}{ab} + \frac{2}{a^2} \right) z, \\ &0 \leq z \leq ab / (a+b). \end{aligned}$$

$$\text{同理: } p_2 f_2(z) = \frac{2}{b} - \left(\frac{2}{ab} + \frac{2}{b^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}.$$

故

$$p_i f_i(z) = \begin{cases} \frac{2}{a} - \left(\frac{2}{ab} + \frac{2}{a^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}, i=1 \\ \frac{2}{b} - \left(\frac{2}{ab} + \frac{2}{b^2} \right) z, 0 \leq z \leq \frac{ab}{a+b}, i=2 \end{cases}.$$

定理 8 若 $(X_1, X_2) \sim BUD_T(a, b), (X'_1, X'_2) \sim BUD_T(a', b')$, 若已知 (Z, I) 与 (Z', I') 同分布, 则所有参数皆可识别.

证明 由 $p_i f_i(z) = p'_i f'_i(z)$, 得

$$\begin{cases} 2/a = 2/a' \\ 2/b = 2/b' \end{cases},$$

得

$$\begin{cases} a = a' \\ b = b' \end{cases}.$$

3 参参数二元均匀分布的识别性

任意三角形的典型形式为: 以原点为顶点, 一边在 X 轴上, 其对应的垂直线平行于 Y 轴, 其顶点为 $P(a, 0)$ 和 $Q(b, c)$ 的三角形 G_2 .

定义 6 称 (X_1, X_2) 服从以原点为顶点, 一边在 X 轴上, 其对应的垂直线平行于 Y 轴, 其顶点为 $P(a, 0)$ 和 $Q(b, c)$ 的三角形 G_2 的二元均匀分布, 其联合密度函数为:

$$f(x, y) = \begin{cases} 2/(ac), (x, y) \in G_2, \\ 0, \text{其他} \end{cases}$$

记作 $(X_1, X_2) \sim BUD_T(a, b, c)$.

引理 9 若 $(X_1, X_2) \sim BUD_T(a, b, c)$, 已知 $b \geq c$, 则 Z 的分布密度为 $g(z) = 2/c - 2z/c^2, 0 \leq z \leq c$.

证明 当 $b \geq c$ 时, Z 的尾概率为:

$$\bar{G}(z) = \int_z^c dy \int_{\frac{by}{c}}^{\frac{b-a}{c}y+a} \frac{2}{ac} dx = \int_z^c (\frac{2}{c} - \frac{2y}{c^2}) dy =$$

$$1 - 2z/c + z^2/c^2, 0 \leq z \leq c,$$

Z 的分布密度为 $g(z) = 2/c - 2z/c^2, 0 \leq z \leq c$.

定理 9 若 $(X_1, X_2) \sim BUD_T(a, b, c), (X'_1, X'_2) \sim BUD_T(a', b', c')$, 已知 $b \geq c, b' \geq c'$, 若已知 Z 与 Z' 同分布, 则仅有参数 c 可识别; 参数 a, b 皆不可识别.

引理 10 若 $(X_1, X_2) \sim BUD_T(a, b, c)$, 已知 $b \geq c$, 则 (Z, I) 的分布密度为:

$$p_i f_i(z) = \begin{cases} 0, 0 \leq z \leq c, i = 1 \\ 2/c - 2z/c^2, 0 \leq z \leq c, i = 2 \end{cases}$$

证明 略.

定理 10 若 $(X_1, X_2) \sim BUD_T(a, b, c), (X'_1, X'_2) \sim BUD_T(a', b', c')$, 已知 $b \geq c, b' \geq c'$, 若已知 (Z, I) 与 (Z', I') 同分布, 则仅有参数 c 可识别; 参数 a, b 皆不可识别.

证明 略.

引理 11 若 $(X_1, X_2) \sim BUD_T(a, b, c)$, 已知 $b < c$, 则 (Z, I) 的分布密度为:

$$p_i f_i(z) = \begin{cases} \frac{2}{(a-b)} + \frac{2(c+a-b)z}{ac(b-a)}, b < z \leq ac/(a+c-b), i = 1, \\ \frac{4(c-b)z}{abc}, 0 \leq z \leq b, i = 1, \\ \frac{2}{c^2} - \frac{2(c+a-b)z}{ac^2}, 0 < z \leq ac/(a+c-b), i = 2. \end{cases}$$

证明 当 $b < c$ 时, 求 (Z, I) 的联合密度:

$$P(Z > z, I = 1) = P(Y > X, X > z) = \int_z^{\frac{ac}{a+c-b}} dx \cdot$$

$$\int_x^{\frac{c}{b-a}(x-a)} \frac{2}{ac} dy = \int_z^{\frac{ac}{a+c-b}} (\frac{2}{(a-b)} + \frac{2(c+a-b)x}{ac(b-a)}) dx = \frac{2}{(a-b)} (\frac{ac}{a+c-b} - z) +$$

$$\frac{(c+a-b)}{ac(b-a)} ((\frac{ac}{a+c-b})^2 - z^2),$$

$$b < z \leq ac/(a+c-b),$$

$$P(Z > z, I = 1) = P(Y > X, X > z) = \bar{G}(z) =$$

$$\int_z^b dx \int_x^{\frac{c}{b-a}x} \frac{2}{ac} dy + \int_b^{\frac{ac}{a+c-b}} dx \int_x^{\frac{c}{b-a}(x-a)} \frac{2}{ac} dy =$$

$$\int_z^b \frac{2(c-b)x}{abc} dx + \int_b^{\frac{ac}{a+c-b}} (\frac{2}{(a-b)} +$$

$$\frac{2(c+a-b)x}{ac(b-a)}) dx = \frac{2(c-b)}{abc} (b^2 - z^2) +$$

$$\frac{2}{(a-b)} (\frac{ac}{a+c-b} - b) + \frac{2(c+a-b)}{ac(b-a)} \cdot$$

$$((\frac{ac}{a+c-b})^2 - b^2), 0 \leq z \leq b$$

$$p_1 f_1(z) = \begin{cases} \frac{2}{(a-b)} + \frac{2(c+a-b)z}{ac(b-a)}, \\ b < z \leq ac/(a+c-b), \\ \frac{4(c-b)z}{abc}, 0 \leq z \leq b. \end{cases}$$

同理:

$$P(Z > z, I = 2) = P(X > Y, Y > z) = \int_z^{\frac{ac}{a+c-b}} dy \cdot$$

$$\int_y^{\frac{b-a}{c}y+a} \frac{2}{ac} dx = \int_z^{\frac{ac}{a+c-b}} (\frac{2}{c^2} - \frac{2(c+a-b)y}{ac^2}) dy =$$

$$\frac{2}{c^2} ((\frac{ac}{a+c-b}) - z) - \frac{(c+a-b)}{ac^2} \cdot$$

$$((\frac{ac}{a+c-b})^2 - z^2).$$

$$p_2 f_2(z) = \frac{2}{c^2} - \frac{2(c+a-b)z}{ac^2}, 0 < z \leq \frac{ac}{a+c-b}.$$

故

$$p_i f_i(z) = \begin{cases} \frac{2}{(a-b)} + \frac{2(c+a-b)z}{ac(b-a)}, \\ b < z \leq ac/(a+c-b), i = 1 \\ \frac{4(c-b)z}{abc}, 0 \leq z \leq b, i = 1 \\ \frac{2}{c^2} - \frac{2(c+a-b)z}{ac^2}, \\ 0 < z \leq ac/(a+c-b), i = 2 \end{cases}.$$

定理 11 若 $(X_1, X_2) \sim BUD_T(a, b, c), (X'_1, X'_2) \sim BUD_T(a', b', c')$, 已知 $b < c, b' < c'$, 若已知 (Z, I) 与 (Z', I') 同分布, 则参数 a, b, c 皆不可识别.

证明 略.

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Identifiability of some bivariate uniform distributions

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Abstract: Identifiability of some bivariate uniform distributions is considered. The result of the study presented in this paper shows: if two random variables have a bivariate uniform distribution of one parameter at the point where the distribution of the minimum is known, then one parameter is identifiable; If two random variables have a bivariate uniform distribution of two parameters, at the point where the distribution of identified minimum is known, then two parameters are identifiable; If two random variables have a bivariate uniform distribution of three parameters, in the case that the distribution of identified minimum is known, then all of parameters are not identifiable.

Key words: bivariate uniform distributions; identifiability; minimum random variable

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