

Dissipation effects on statistical correlation in Moshinsky atom

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Abstract: We investigate the dissipation effects on the statistical correlation for one-dimension(1D) Moshinsky atom. Using the method developed by Yu and Sun, the general analytical expressions extracted from the information theory such as Shannon entropies, correlation coefficient and mutual information in position and momentum are obtained, and their behaviors as functions of time and the strength of the interparticle interaction are demonstrated. Contrary to the usual case in which there exist two effects in dissipative systems, it shows that only the damping plays the important role while the effects of Brownian motion are little significant.

Key words: dissipative Moshinsky atom; Shannon entropy; correlation coefficient; mutual information; Brownian motion

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The study of correlation is of both conceptual and technological interest because it is a key concept in understanding various chemical and physical phenomena. Especially, it is thought to be responsible for many interesting quantum phenomena such as entanglement^[1-3]. The non-commutativity of some operators is an important source of statistical correlation in quantum mechanics, which introduces the uncertainty relation and exists in systems of one or many particles, and can be quantified in terms of correlation coefficients^[4]. Furthermore, taken from information theory^[5-6], there exists a more general form of correlation^[7-11], known as mutual information^[5] which is based on extensively studied Shannon entropies^[10-16].

On the other hand, due to the fact that it is analytically solvable, the Moshinsky atom^[17] has been the test ground for various theoretical methods in many areas of physics^[17-22]. The study on statistical correlations in the Moshinsky atom via the correlation

coefficient and mutual information in position and momentum space has also been done^[10]. Since the system is always influenced by the environment, the work was extended to the case when the dissipation^[23] is present^[24], and the Lindblad form of the Markovian master equation^[25] was numerically solved and the Shannon entropies were plotted.

There are two main approaches for the dissipative quantum systems, with the first one being the system plus bath approach. This approach will provide both the friction and the fluctuation force, and two effects occur for classical and quantum systems. The first is dissipation due to damping, and the second is the fluctuating force exerted by the environment on the system which manifests itself for example as Brownian motion. The second approach is the use of an effective Hamiltonian, which yields the dissipation equation through its Heisenberg equation. The model of quantum damped harmonic oscillator^[26-27] is a typical example

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that results from the application of the effective Hamiltonian approach to the dissipative harmonic oscillator while ignoring the effect of the Brownian motion. The connection between the effective Hamiltonian and the system plus bath approach has been shown by Yu and Sun^[28-29]. They obtained an exact solution for the wave function of the system plus the bath.

In the present work, we investigate the Moshinsky atom in the presence of dissipation by applying the YS method. Different from the work of Ref. [24], the system is initially assumed to be in the ground state, and the Shannon entropies, correlation coefficients and mutual information are all calculated analytically.

1 Model and calculation

Let us begin by the following Hamiltonian of the dissipative Moshinsky atom^[30],

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_I, \quad (1)$$

where

$$\hat{H}_S = \frac{\hat{p}_1^2 + \hat{p}_2^2}{2m} + \frac{1}{2}m\omega_0^2(x_1^2 + x_2^2) + \frac{1}{2}m\omega_0^2\kappa(x_1 - x_2)^2, \quad (2)$$

which is the Hamiltonian of the Moshinsky atom, describing two harmonically interacting electrons trapped in a 1D harmonic oscillator potential,

$$\hat{H}_B = \sum_j^{N_B} \left(\frac{\hat{p}_j^2}{2m_j} + \frac{1}{2}m_j\omega_j^2 q_j^2 \right), \quad (3)$$

which is the Hamiltonian of the bath, comprised of N_B harmonic oscillators B_i with mass m_i , coordinates X_i and frequency ω_i , and

$$\hat{H}_I = \left(\sum_j^{N_B} c_j q_j \right) (x_1 + x_2), \quad (4)$$

which is the interaction between the system and bath. It is evident that Hamiltonian of the system $\hat{H}_S$ can be decomposed into the center-of-mass (CM) and relative motion,

$$\hat{H}_S = \hat{H}_{\text{cm}} + \hat{H}_{\text{rel}}, \quad (5)$$

with

$$\hat{H}_{\text{cm}} = \frac{\hat{P}^2}{2M} + \frac{1}{2}M\omega_0^2 X^2, \quad \hat{H}_{\text{rel}} = \frac{\hat{p}^2}{2\mu} + \frac{1}{2}\mu\Omega^2 x^2, \quad (6)$$

which are being the Hamiltonian of CM and relative motion, respectively. Notice here, the relative motion coordinates $\hat{p} = (\hat{p}_1 - \hat{p}_2)/2$, $x = x_1 - x_2$, with reduced mass $\mu = m/2$ and frequency $\Omega = \omega_0\sqrt{2\kappa+1}$, while the CM motion coordinates $\hat{P} = \hat{p}_1 + \hat{p}_2$, $X = (x_1 + x_2)/2$ with total mass $M = 2m$. Different from Ref. [24], we assume initially the system is in the ground state of the Moshinsky atom, i.e., at time $t=0$, $\Psi_0(x_1, x_2) = \varphi_0(x)\Phi_0(X)$, with

$$\varphi_0(x) = \left(\frac{\mu\Omega}{\pi\hbar} \right)^{1/4} e^{-\frac{\mu\Omega}{2\hbar}x^2},$$

$$\Phi_0(X) = \left(\frac{M\omega_0}{\pi\hbar} \right)^{1/4} e^{-\frac{M\omega_0}{2\hbar}X^2}, \quad (7)$$

which are the ground states of the relative and CM part of the Hamiltonian. Since the bath is coupled only to the CM motion, according to the YS method, we have the wave function at time t ,

$$\Psi(x_1, x_2, t) = \varphi_0(x)\Phi_0(X - \xi, t), \quad (8)$$

with $\xi = \sum_j \xi_j$ being the Brownian motion or the fluctuation of the system coordinates which is proportional to the coupling strength between the CM motion of the system and the bath, and obeys the Gaussian distribution^[29]. Employing the Feynman's path integral method^[31], we obtain

$$\Phi_0(X - \xi; t) = \frac{N_0 e^{\frac{1}{4}\eta t - A(t)(X - \xi)^2 - \frac{1}{2}\text{arccot}\left[\frac{\omega_0}{\omega}\left(\frac{\eta}{2\omega} + \cot\omega t\right)\right]}}{\sqrt{|\zeta(t)\sin\omega t|}}, \quad (9)$$

with

$$\omega = \sqrt{\omega_0^2 - \eta^2/4}, \quad \alpha_0 = \sqrt{(M\omega_0)/\hbar},$$

$$N_0 = (\alpha_0/\sqrt{\pi})^{1/2}, \quad \alpha = \sqrt{(M\omega)/\hbar},$$

$$A(t) = \frac{\alpha^2 e^{\eta t}}{2} \left[\frac{\zeta(t)}{|\zeta(t)^2 \sin^2 \omega t|} - i(\cot \omega t - \frac{\eta}{2\omega}) \right],$$

$$\zeta(t) = \frac{\omega_0}{\omega} + i\left(\frac{\eta}{2\omega} + \cot \omega t\right).$$

We refer the readers to Ref. [32] for more details of the calculations of Eq. (9). Note that in the limit of $\eta \rightarrow 0$ or t approaches 0, the wave function of Eq. (8)

reduces to that of the ground state of the Moshinsky atom as it must. Next, we use the wave functions of Eq.(8) to investigate the statistical correlations by employing the mutual information, together with localization properties to perform the correlation analysis on phase-space distribution.

2 Result and discussion

We first perform the localization measures using the Shannon entropies. Analytical expressions for pair Shannon entropies in position and momentum space can be obtained as

$$s_F(t) = -\int dx_1 dx_2 \Gamma(x_1, x_2) \ln \Gamma(x_1, x_2) = 1 + \ln \pi - \ln \left(\frac{(M \mu \Omega \omega_0 e^{\eta t})^{1/2}}{\hbar |\zeta(t) \sin \omega t|} \right), \quad (10)$$

$$s_{\Pi}(t) = -\int dp_1 dp_2 \Pi(p_1, p_2) \ln \Pi(p_1, p_2) = 1 + \ln \pi + \frac{1}{2} \ln \left(\frac{\hbar^3 \mu \Omega (4B^2 + C^2)}{4B} \right), \quad (11)$$

where

$$B = \frac{\alpha^2 e^{\eta t} \omega_0}{\omega |\zeta(t)|^2 \sin^2 \omega t}, \quad C = 2\alpha^2 e^{\eta t} \left(\frac{\eta / (2\omega) + \cot \omega t}{|\zeta(t)|^2 \sin^2 \omega t} - (\cot \omega t - \frac{\eta}{2\omega}) \right), \quad (12)$$

with

$$\Gamma(x_1, x_2) = |\Psi(x_1, x_2, t)|^2, \quad \Pi(p_1, p_2) = |\Phi(p_1, p_2, t)|^2$$

being the pair densities momentum space defined in terms of the wave functions of Eq.(8) in their momentum space representations^[11]. As plotted in the upper panel of Fig.1, dissipation causes $s_F(t)$ decreases while $s_{\Pi}(t)$ increases wavyly with time at fixed values of κ , and their sum $s_F(t) + s_{\Pi}(t)$ fluctuates about a constant value. In the lower panel of Fig.1, we plot $s_F(t)$, $s_{\Pi}(t)$ and $s_F(t) + s_{\Pi}(t)$ as functions of κ at different values of time. It is clear that as the value of κ increases, $s_F(t)$ decreases while $s_{\Pi}(t)$ increases monotonically, but their sum remains a constant. These behaviors are very similar to the case where the dissipation is absent as shown in Ref. [10],

meaning the position space pair distribution localizes as time increases while the momentum space pair has the opposite behavior. This may be interpreted as a consequence of the uncertainty-type relationship, i.e. the sum

$$s_F(t) + s_{\Pi}(t) = 2(1 + \ln \pi) + \frac{1}{2} \ln \left[\hbar^4 \left(1 + \frac{C^2}{4B^2} \right) \right] \geq 2(1 + \ln \pi), \quad (13)$$

which satisfies entropic uncertainty relation at the two-particle level in atomic units ($\hbar=m=1$). It is interesting to note that the sum $s_F(t) + s_{\Pi}(t)$ also does not depend upon κ . This means the localization in position space is perfectly compensated with an equal delocalization in momentum space, the bath has the same effect in both spaces at any time irrespective of the value of κ .

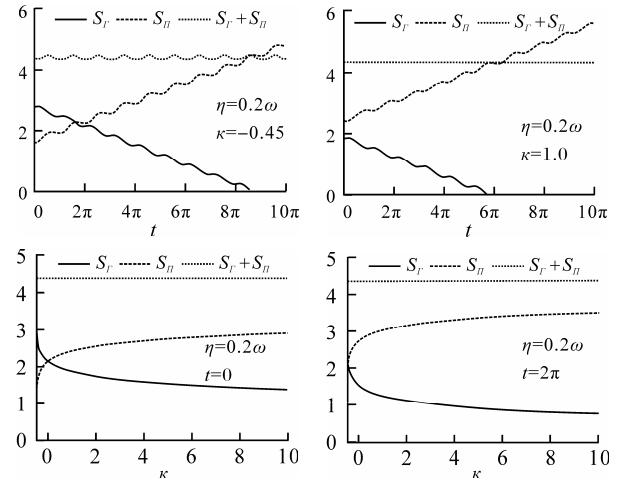


Fig. 1 $s_F(t)$, $s_{\Pi}(t)$ as functions of t and κ

On the other hand, the one-particle Shannon entropies in position and momentum space can also be obtained analytically,

$$s_{\rho}(t) = -\int dx \rho(x) \ln \rho(x) = \frac{1}{2} (1 + \ln \pi) - \frac{1}{2} \ln \left[\frac{4\mu\Omega\omega_0\alpha^2}{4\omega\mu\Omega|\zeta(t)|^2 \sin^2 \omega t + \hbar\omega_0\alpha^2 e^{\eta t}} \right], \quad (14)$$

$$s_{\pi}(t) = -\int dp \pi(p) \ln \pi(p) = \frac{1}{2} (1 + \ln \pi) + \frac{1}{2} \ln \left[\hbar\mu\Omega + \frac{\hbar^2 B}{4} \left(1 + \frac{C^2}{4B^2} \right) \right], \quad (15)$$

with

$$\rho(x) = \int dx_2 \Gamma(x_1, x_2), \quad \pi(p) = \int dp_2 \Pi(p_1, p_2)$$

are the one-particle densities in position and momentum

spaces. In Fig. 2, we plot one-particle Shannon entropies S_p and S_π as functions of time for $\kappa = -0.45, 1.0$, and as functions of κ at time $t=0, 2\pi$ for confining potentials $\omega_0=1.0, \eta=0.2\omega_0$. It is clear that dissipation causes S_π to increase wavy as time t increases meaning the one-particle distribution delocalizes in momentum space, and S_p to decrease but more slowly meaning a weaker localization in the position space distribution (information gain), leading to the wavyly increasing behavior of their sum S_p+S_π as a function of time. Moreover, S_π increases while S_p decreases as the values of κ increases as functions of κ , and their sum

$$s_p(t) + s_\pi(t) = (1 + \ln \pi) + \frac{1}{2} \ln \left(\frac{\pi \left(\hbar \mu \Omega + \frac{\hbar^2 B}{4} \left(1 + \frac{\hbar^2 C^2}{16 B^2} \right) \right)}{4 \mu \Omega \omega_0 \alpha^2} \right) \geq (1 + \ln \pi) \quad (16)$$

always satisfies the entropic uncertainty relation at the one-particle level in atomic units. It is worthy pointing out that in the limit of $\eta \rightarrow 0$, the above results of Eqs. (5)-(16) reduce to those in the absence of dissipation^[10].

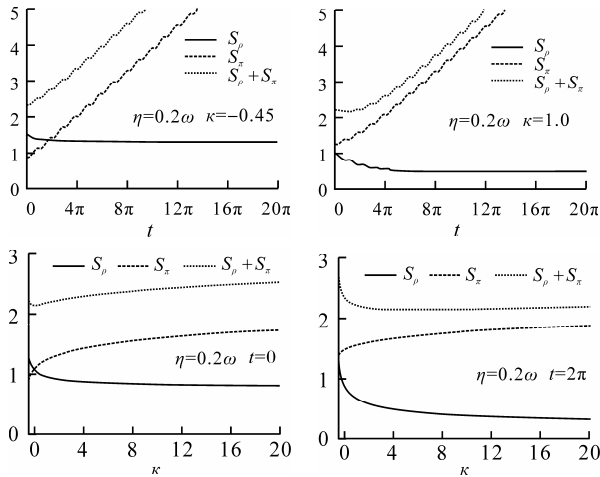


Fig. 2 S_p and S_π as functions of t and κ

Next we perform the correlation measures, that is based on the correlation coefficients in position and momentum spaces, which are defined as

$$\sigma_x = \frac{\langle x_1 x_2 \rangle - \langle x \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2}, \quad \sigma_p = \frac{\langle p_1 p_2 \rangle - \langle p \rangle^2}{\langle p^2 \rangle - \langle p \rangle^2}, \quad (17)$$

with the moments of the one-particle being defined as

$$\langle x \rangle = \int dx x \rho(x), \quad \langle p \rangle = \int dp p \pi(p),$$

and two-particle densities are defined as

$$\langle x_1 x_2 \rangle = \int dx_1 dx_2 x_1 x_2 \Gamma(x_1, x_2), \quad \langle p_1 p_2 \rangle = \int dp_1 dp_2 p_1 p_2 \Pi(p_1, p_2).$$

After some algebra manipulations, we obtain

$$\sigma_x = \frac{4 \mu \Omega \omega |\zeta(t)^2 \sin^2(\omega t) - \hbar \alpha^2 e^{\eta t} \omega_0}{4 \mu \Omega \omega |\zeta(t)^2 \sin^2(\omega t) + \hbar \alpha^2 e^{\eta t} \omega_0},$$

$$\sigma_p = \frac{-16 \mu \Omega \hbar B + 4 \hbar^2 B^2 + \hbar^2 C^2}{16 \mu \Omega \hbar B + 4 \hbar^2 B^2 + \hbar^2 C^2}. \quad (18)$$

It can be checked that in the absence of dissipation $\eta \rightarrow 0$, Eq. (18) reduces to those obtained in Ref. [10].

In Fig. 3, we plot σ_x and σ_p as functions of t at three

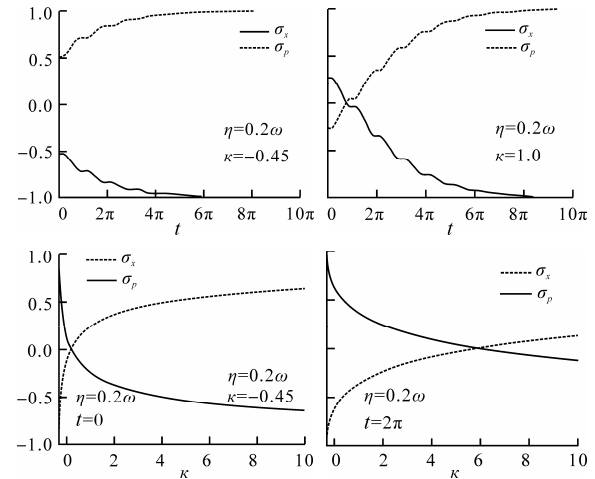


Fig. 3 σ_x and σ_p as functions of t and κ

different values of $\kappa = -0.45, 1.0$ (upper panel), and as functions of κ at different time $t=0, 2\pi$ (lower panel). It is evident that dissipation causes σ_x decreases wavy and approaches -1 as t increases, while σ_p has the opposite behavior. At fixed time, σ_x decreases while σ_p increases monotonically as the value of κ increases. These results are similar to those obtained in the absence of dissipation as shown in Ref. [10].

Mutual information in position and momentum spaces can be calculated as

$$I_x = \int dx_1 dx_2 \Gamma(x_1, x_2) \ln \left(\frac{\Gamma(x_1, dx_2)}{\rho(x_1) \rho(x_2)} \right) = \ln \left(\frac{\sqrt{e^{\eta t}} (\Omega |\zeta(t)^2 \sin^2(\omega t) + \omega_0 e^{\eta t})}{2 \sqrt{\Omega \omega_0} |\zeta(t) \sin(\omega t)|} \right),$$

$$I_p = \int dp_1 dp_2 \Pi(p_1, p_2) \ln \left(\frac{\Pi(p_1, p_2)}{\pi(p_1)\pi(p_2)} \right) = \ln \left(\frac{16\mu\Omega B + \hbar(4B^2 + C^2)}{8\sqrt{\hbar\mu\Omega B(4B^2 + C^2)}} \right). \quad (19)$$

It is clear from Eq. (19) that I_x does not depend upon the confining potential ω_0 while I_p does. From the upper panel of Fig. 4, it is clear that the statistical correlation between particles increases as time goes on. Thus, the effect of the bath makes the system more correlated at fixed value of κ . In the lower panel of Fig. 4, we plot the mutual information I_x and I_p as functions of

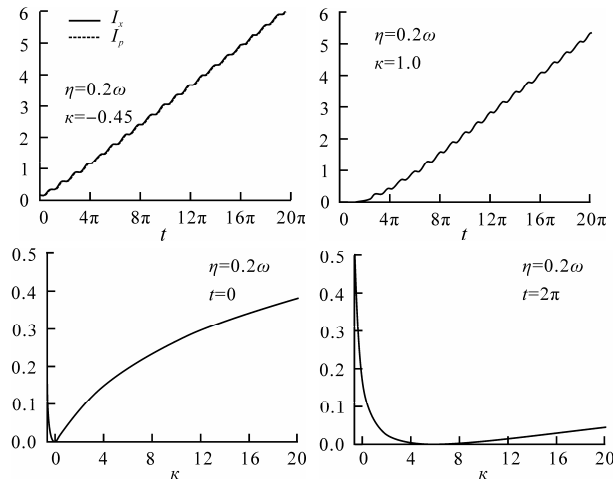


Fig. 4 I_x and I_p as functions t and κ

κ at two fixed values if time t . We can see that as the value of κ increases, the statistical correlation first decays to 0, but as the value of κ continues to increase, the values of I_x and I_p start to increase monotonically, i.e. the system becomes more correlated as the interaction between the two particles becomes more attractive.

3 Conclusion

In summary, we have investigated the dissipation effects on the statistical correlation for the dissipative Moshinsky atom. Using the YS method, the general analytical expressions for the Shannon entropies, correlation coefficient and mutual information in position and momentum spaces are obtained. Their behaviors as functions of time and the strength of the interparticle interaction are discussed. It is shown that

for the two effects that usually occur in dissipative systems, only the damping places an important role while the effects of Brownian motion disappeared.

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耗散效应对 Moshinsky 原子统计关联的影响

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摘要: 研究了耗散效应对一维 Moshinsky 原子统计关联的影响. 利用 Yu 和 Sun 的方法, 得到了关于香农熵、关联函数和位置及动量中的交互信息等信息理论的一般解析表达式, 且给出了这些函数随时间和粒子间作用强度变化的图像. 与通常耗散系统中存在 2 种效应相反, 研究结果显示, 在本系统中只有阻尼起到重要效应, 而布朗运动效应消失.

关键词: 耗散 Moshinsky 原子; 香农熵; 关联系数; 交互信息; 布朗运动

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