

# Atomic martingale decompositions of two-parameter $B$ -valued martingales

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**Abstract:** The introduction of two-parameter  $B$ -valued atomic martingale is followed by establishing atomic martingale decompositions of two-parameter  $B$ -valued martingales in Hardy spaces  ${}_p H_\alpha^s(B)$  and  ${}_p H_\alpha^s(B)$ , as defined by the  $p$ -variation function and conditional  $p$ -variation function respectively. Different from the results of the atomic function, those obtained here do not depend on the geometrical properties of the Banach space, and extend the corresponding known results from one-parameter setting to two-parameter setting.

**Key words:** two-parameter  $B$ -valued martingale; atomic martingale; atomic decomposition

**CLC number:** O211.6

**Document code:** A

**Article ID:** 1001-5132 ( 2019 ) 05-0095-05

## 1 Introduction

The method of atomic decomposition, introduced by Herz<sup>[1]</sup> and Bernard<sup>[2]</sup> in the martingale setting and developed by Weisz<sup>[3]</sup>, is a key ingredient in dealing with many problems including martingale inequalities, duality, interpolation and so on. Because the technique of stopping times used in studying one-parameter martingales is usually unsuitable for two-parameter martingales, whereas the method of atomic decomposition can be used in both cases, hence this method plays an important role in martingale theory, especially for multi-parameter martingale and small-index martingale.

In the scalar-valued case<sup>[3]</sup>, atoms are usually defined as functions. However, it is well known that the existence of the decompositions by atomic functions essentially depends on the geometrical properties of Banach space in which the martingales take values in

the vector-valued case<sup>[4-9]</sup>. From the point of view of modern mathematics, martingales are more capable of reflecting the process, information and approximation than functions. Recently, enlightened by Martínez and Torrea<sup>[10]</sup>, Yu<sup>[11-13]</sup> introduced the  $B$ -valued atomic martingale and studied the atomic decomposition in one-parameter  $B$ -valued martingale spaces. The results eliminated the restrictions on the geometrical properties of Banach spaces of the existence of the decompositions. Therefore, based on a slight generalization of atomic function, the authors think that it is more suitable to define the atoms as martingales than functions.

The main purpose of this paper is to study atomic martingale decompositions of two-parameter  $B$ -valued martingales in Hardy spaces  ${}_p H_\alpha^s(B)$  defined by the  $p$ -variation function and  ${}_p H_\alpha^s(B)$  defined by the conditional  $p$ -variation function of two-parameter  $B$ -valued martingales. The results extend atomic decompositions in [10-11] to two-parameter  $B$ -valued martingale setting.

## 2 Preliminaries and notations

Let  $(\Omega, F, P)$  be a complete probability space and  $B$  denote a Banach space (real separable Banach space) with norm  $\|\cdot\|$ . Let  $\{F_n, n \in N^2\}$  be a family of non-decreasing sub- $\sigma$ -algebras of  $F$  with respect to the partial ordering on  $N^2$  such that  $F = \sigma(\bigcup_{n \in N^2} F_n)$ . Set  $F_{n-1} = F_{n_1-1, n_2-1}$ , the expectation operator is denoted by  $E$ , and conditional expectation operators relative to  $F_n$  and  $F_{n-1}$  are denoted by  $E_n$  and  $E_{n-1}$ .

An integrable  $B$ -valued sequence of functions  $f = (f_n, n \in N^2)$  is said to be a martingale if it is adapted to  $\{F_n, n \in N^2\}$  and  $E_n f_m = f_n$  for all  $n \leq m$ .

We define the sequence of martingale differences  $df = (df_n, n \in N^2)$  as follows:

$$df_n = \begin{cases} 0, & n_1 n_2 = 0, \\ f_{n_1, n_2} - f_{n_1-1, n_2} - f_{n_1, n_2-1} + f_{n_1-1, n_2-1}, & n_1 n_2 \neq 0. \end{cases}$$

And set

$$F_{-1} = F_{-1, i} = F_{i, -1} = F_0 = \{\phi, \Omega\},$$

$$f_{-1} = f_{-1, i} = f_{i, -1} = f_0 = 0.$$

The stochastic basis  $\{F_n, n \in N^2\}$  is said to be regular if there exists a number  $R > 0$  such that  $f_{n_1, n_2} \leq R f_{n_1-1, n_2}$ ,  $f_{n_1, n_2} \leq R f_{n_1, n_2-1}$  for all non-negative real-valued martingale  $f = (f_n, n \in N^2)$ . It can be shown that if  $\{F_n, n \in N^2\}$  is regular then for all  $p > 0$  there exists  $R_p > 0$  such that  $\|d_n f\|^p \leq R_p E_{n-1} \|d_n f\|^p$  for every  $B$ -valued martingale  $f = (f_n, n \in N^2)$ .

A function  $\nu$  which maps  $\Omega$  into the set of subspaces of  $N^2 \cup \{\infty\}$  is said to be a two-parameter stopping time relative to  $\{F_n, n \in N^2\}$  if for all  $\omega \in \Omega$ ,  $\nu(\omega)$  are incomparable and for any  $n \in N^2$ ,  $\{\omega \in \Omega : n \in \nu(\omega)\} \in F_n$ . The set of all two-parameter stopping times will be denoted by  $T_2$ . It is easy to see that  $\nu \in T_2$  iff  $\{\nu \leq n\} \in F_n$  for all  $n \in N^2$ . Furthermore,  $\{\nu \leq n\} \in F_{n-1}$  ( $n \in N^2$ ) if  $\nu \in T_2$ .

For a martingale  $f = (f_n, n \in N^2)$  and stopping time  $\nu$ , the stopped martingale  $f^{(\nu)} = (f_n^{(\nu)}, n \in N^2)$  is defined by

$$f_n^{(\nu)} = \sum_{m \leq n} \chi(\nu \leq m) d_m f, \quad n \in N^2.$$

For a two-parameter  $B$ -valued martingale  $f = (f_n, n \in N^2)$  and  $1 \leq p < \infty$  we denote by  $f^*$ ,  $S^{(p)}(f)$ ,  $s^{(p)}(f)$  and  $\|f\|_p$  its Maximal function,  $p$ -variation function, conditional  $p$ -variation function and  $L_p$ -norm as follows:

$$f_n^* = \sup_{i \leq n} \|f_i\|, \quad f^* = \sup_{n \in N^2} \|f_n\|,$$

$$S_n^{(p)}(f) = \left( \sum_{0 \leq i \leq n} \|d_i f\|^p \right)^{1/p},$$

$$S^{(p)}(f) = \sup_{n \in N^2} S_n^{(p)}(f), \quad 0 < p < \infty,$$

$$s_n^{(p)}(f) = \left( \sum_{0 \leq i \leq n} E_{i-1} \|d_i f\|^p \right)^{1/p},$$

$$s^{(p)}(f) = \sup_{n \in N^2} s_n^{(p)}(f),$$

$$\|f\|_p = \sup_{n \in N^2} \|f_n\|_p.$$

Then, for  $0 < \alpha \leq \infty$ , we define Hardy spaces of two-parameter  $B$ -valued martingales by

$$H_\alpha^*(B) = \{f : \|f\|_{H_\alpha^*(B)} = \|f^*\|_\alpha < \infty\},$$

$${}_p H_\alpha^S(B) = \{f : \|f\|_{{}_p H_\alpha^S(B)} = \|S^{(p)}(f)\|_\alpha < \infty\},$$

$${}_p H_\alpha^s(B) = \{f : \|f\|_{{}_p H_\alpha^s(B)} = \|s^{(p)}(f)\|_\alpha < \infty\}.$$

In this paper, we denote by  $C$  the constants which can take different values in different places.

## 3 Atomic martingale decompositions

In this section we formulate several atomic martingale decomposition theorems of the spaces  ${}_p H_\alpha^S(B)$  and  ${}_p H_\alpha^s(B)$ .

**Definition 1** A two-parameter  $B$ -valued martingale  $a = (a_n, n \in N^2)$  is called an atomic martingale of the type  $A^{s(p)}(\alpha, \infty)$  (or  $A^{S(p)}(\alpha, \infty)$ ) ( $1 < p < \infty$ ,  $0 < \alpha < \infty$ ), if there is a corresponding stopping time  $\nu \in T_2$ , such that

$$(i) \quad a_n = 0, \text{ if } \nu \leq n, \forall n \geq 0;$$

$$(ii) \quad \|s^{(p)}(a)\|_\infty \leq P(\nu \neq \infty)^{-1/\alpha}, \text{ or}$$

$$(ii)' \quad \|S^{(p)}(a)\|_\infty \leq P(\nu \neq \infty)^{-1/\alpha}.$$

**Definition 2** A  $B$ -valued integrable function  $a$  is called an atomic function of the type  $A^{s(p)}(\alpha, \infty)$  (or  $A^{S(p)}(\alpha, \infty)$ ) ( $1 < p < \infty$ ,  $0 < \alpha < \infty$ ), if there is a corresponding stopping time  $\nu \in T_2$ , such that

- (i)  $a_n \triangleq E_{n-1}a = 0$ , if  $v \ll n, \forall n \geq 1$ ;
- (ii)  $\|s^{(p)}(a)\|_\infty \leq P(v \neq \infty)^{-1/\alpha}$ , or
- (ii)'  $\|S^{(p)}(a)\|_\infty \leq P(v \neq \infty)^{-1/\alpha}$ .

**Remark** Quite evidently, an atomic function is an atomic martingale, but not vice versa. Definition 1 is a slight generalization of Definition 2. The atomic function decomposition theorems of the spaces  ${}_p H_\alpha^s(B)$  and  ${}_p H_\alpha^s(B)$  can be seen in [6-7].

**Theorem 1** Let  $1 < p < +\infty$ ,  $0 < \alpha \leq p$ ,  $B$  is a Banach space. Then, for every two-parameter  $B$ -valued martingale  $f = (f_n, n \in N^2) \in {}_p H_\alpha^s(B)$ , there exists a sequence of type  $A^{s(p)}(\alpha, \infty)$  atomic martingales  $\{a^{(k)} = (a_n^{(k)}, n \in N^2)\}_{k \in Z}$  with the corresponding stopping time  $v_k$  and a sequence of positive numbers  $\{\mu_k\}_{k \in Z} \in l_\alpha$ , such that for all  $n \in N^2$

$$f_n = \sum_{k \in Z} \mu_k a_n^{(k)}, \text{ a.e.}, \quad (1)$$

and

$$\|f\|_{{}_p H_\alpha^s(B)} \approx \inf \left( \sum_{k \in Z} \mu_k^\alpha \right)^{1/\alpha} < \infty, \quad (2)$$

where the infimum is taken over all such decompositions of  $f$  of the form (1). Furthermore, all of the atomic martingales  $\{a^{(k)} = (a_n^{(k)}, n \in N^2)\}_{k \in Z}$  belong to the space  ${}_p H_\alpha^s(B)$  too.

**Proof** Let two-parameter  $B$ -valued martingale  $f = (f_n, n \in N^2) \in {}_p H_\alpha^s(B)$ . It is easy to check that  $s^{(p)}(f) < \infty$ , a.e.. Set  $F_k = \{s^{(p)}(f) > 2^k\}$  and  $v_k = \inf\{n \in N^2 : E_n \chi(F_k) > 1/2\}$ ,  $(\inf\{\emptyset\} = \infty)$ .

Obviously, as  $k \rightarrow \infty$ , stopping time  $v_k \rightarrow \infty$ .  $v_k$  is non-decreasing and  $F_k$  is non-increasing with respect to  $k$ , respectively. Set

$$f_n^{(v_k)} = \sum_{m \leq n} \chi(v_k \ll m) d_m f.$$

It is easy to see that  $(f_n^{(v_k)}, n \in N^2)$  is a stopped martingale and

$$f_n^{(v_{k+1})} - f_n^{(v_k)} = \sum_{m \leq n} \chi(v_k \ll m \not\ll v_{k+1}) d_m f.$$

Now we prove that for all  $n \in N^2$

$$f_n = \sum_{k \in Z} (f_n^{(v_{k+1})} - f_n^{(v_k)}), \text{ a.e.} \quad (3)$$

Denote

$$(U) = \sum_{n \in N^2} E(\chi(s^{(p)}(f) \leq 2^{k+1}) E_{n-1}(\|d_n f\|^p \cdot \chi(v_k \ll n \not\ll v_{k+1}))),$$

$$(V) = \sum_{n \in N^2} E(\chi(s^{(p)}(f) > 2^{k+1}) E_{n-1}(\|d_n f\|^p \cdot \chi(v_k \ll n \not\ll v_{k+1}))).$$

Obviously,

$$(U) \leq 2^{p(k+1)} P(v_k \neq \infty),$$

$$(V) = \sum_{n \in N^2} E(E_{n-1} \chi(F_{k+1}) E_{n-1}(\|d_n f\|^p \cdot \chi(v_k \ll n \not\ll v_{k+1}))).$$

From the definition of  $v_{k+1}$  it follows  $E_{n-1} \chi(F_{k+1}) \leq 1/2$  when  $v_{k+1} \ll n$ .

So

$$(V) \leq \frac{1}{2} \sum_{n \in N^2} E(E_{n-1}(\|d_n f\|^p \chi(v_k \ll n \not\ll v_{k+1}))) =$$

$$\frac{1}{2} \sum_{n \in N^2} E(\|d_n f\|^p \chi(v_k \ll n \not\ll v_{k+1})).$$

Therefore

$$\sum_{n \in N^2} E(\|d_n f\|^p \chi(v_k \ll n \not\ll v_{k+1})) = (U) + (V) \leq$$

$$2^{p(k+1)} P(v_k \neq \infty) + \frac{1}{2} \sum_{n \in N^2} E(\|d_n f\|^p \cdot \chi(v_k \ll n \not\ll v_{k+1})),$$

and

$$E\left(\sum_{n \in N^2} \|d_n f\|^p \chi(v_k \ll n \not\ll v_{k+1})\right) \leq$$

$$2 \cdot 2^{p(k+1)} P(v_k \neq \infty).$$

Similarly, we can get

$$E\left(\sum_{n \in N^2} \|d_n f\|^p \chi(v_l \ll n)\right) \leq 2 \cdot 2^{pl},$$

which implies

$$\lim_{l \rightarrow -\infty} E\left(\sum_{m \leq n} \|d_m f\|^p \chi(v_l \ll m)\right) = 0.$$

Noticing that  $\chi(v_l \ll m)$  is decreasing with respect to  $l$ , we get

$$\lim_{l \rightarrow -\infty} \sum_{m \leq n} \|d_m f\|^p \chi(v_l \ll m) = 0, \text{ a.e.},$$

and

$$\lim_{l \rightarrow -\infty} \sum_{m \leq n} \|d_m f\| \chi(v_l \ll m) = 0, \text{ a.e.},$$

which implies  $\lim_{m \rightarrow -\infty} f_n^{(v_l)} = 0$  a.e. Because  $\lim_{m \rightarrow \infty} f_n^{(v_{m+1})} = f_n$  a.e. and  $\sum_{k=l}^{v_{k+1}} (f_n^{(v_{k+1})} - f_n^{(v_k)}) = f_n^{(v_{m+1})} - f_n^{(v_l)}$ , which proves Eq. (3).

Set

$$\mu_k = 3 \times 2^k P(v_k < \infty)^{1/\alpha},$$

$$a_n^{(k)} = \frac{1}{\mu_k} (f_n^{(v_{k+1})} - f_n^{(v_k)}).$$

Then  $\{a^{(k)} = (a_n^{(k)}, n \in N^2)\}$  is a two-parameter  $B$ -valued martingale for any fixed  $k \in Z$ .

From  $s^{(p)}(f^{(v_k)}) = s_{v_k}^{(p)}(f) \leq 2^k$ , we have

$$\begin{aligned} s^{(p)}(a^{(k)}) &= \left( \sum_{n \in N^2} E_{n-1} \|d_n a^{(k)}\|^p \right)^{\frac{1}{p}} = \\ &= \frac{1}{\mu_k} \left( \sum_{n \in N^2} E_{n-1} \|f_n^{(v_{k+1})} - f_n^{(v_k)}\|^p \right)^{\frac{1}{p}} \leq \\ &= \frac{1}{\mu_k} (s^{(p)}(f_n^{(v_{k+1})}) + s^{(p)}(f_n^{(v_k)})) \leq \\ &= \frac{1}{\mu_k} (2^{k+1} + 2^k) = \frac{2^{k+1} + 2^k}{3 \times 2^k} P(v_k < \infty)^{-\frac{1}{\alpha}} = \\ &= P(v_k < \infty)^{-\frac{1}{\alpha}}. \end{aligned}$$

Moreover,  $\|s^{(p)}(a^{(k)})\|_\infty \leq P(v_k < \infty)^{-1/\alpha}$ , which means that  $a^{(k)} = (a_n^{(k)}, n \in N^2)$  for all  $k \in Z$  are  $A^{s(p)}(\alpha, \infty)$  atomic martingales. By the definition of  $\mu_k$  and  $a_n^{(k)}$ , from Eq. (3), we have  $f_n = \sum_{k \in Z} \mu_k a_n^{(k)}$ , a.e., for all  $n \in N^2$ .

Then Eq. (1) is proved.

To estimate  $\sum_{k \in Z} \mu_k^\alpha$ , we can use Abel's transformation,

$$\begin{aligned} \sum_{k \in Z} \mu_k^\alpha &= 3^\alpha \sum_{k \in Z} 2^{k\alpha} P(v_k < \infty) = \\ &= 3^\alpha \sum_{k \in Z} 2^{k\alpha} P(s^{(p)}(f) > 2^k) = \\ &= \frac{3^\alpha}{2^\alpha - 1} \sum_{k \in Z} (2^{(k+1)\alpha} - 2^{k\alpha}) P(s^{(p)}(f)^\alpha > 2^{k\alpha}) = \\ &= \frac{3^\alpha}{2^\alpha - 1} \sum_{k \in Z} 2^{k\alpha} P(2^{(k-1)\alpha} < s^{(p)}(f)^\alpha \leq 2^{k\alpha}) = \\ &= \frac{3^\alpha \cdot 2^\alpha}{2^\alpha - 1} E[s^{(p)}(f)^\alpha] = \\ &= \frac{6^\alpha}{2^\alpha - 1} \|f\|_{p, H_\alpha^s(B)}^\alpha < \infty. \end{aligned} \quad (4)$$

We also have

$$\begin{aligned} \sum_{k \in Z} \mu_k^\alpha &= \frac{3^\alpha}{2^\alpha - 1} \sum_{k \in Z} 2^{k\alpha} P(2^{(k-1)\alpha} < s^{(p)}(f)^\alpha \leq 2^{k\alpha}) \geq \\ &= \frac{3^\alpha}{2^\alpha - 1} E[s^{(p)}(f)^\alpha] = \frac{3^\alpha}{2^\alpha - 1} \|f\|_{p, H_\alpha^s(B)}^\alpha. \end{aligned} \quad (5)$$

Combining Eq. (4) and (5), we gain that there exist two constants  $C_1 > 0$  and  $C_2 > 0$ , such that

$$C_1 \inf_{k \in Z} \left( \sum_{k \in Z} \mu_k^\alpha \right)^{\frac{1}{\alpha}} \leq \|f\|_{p, H_\alpha^s(B)} \leq C_2 \inf_{k \in Z} \left( \sum_{k \in Z} \mu_k^\alpha \right)^{\frac{1}{\alpha}} < \infty.$$

Then Eq. (2) is obtained.

Furthermore, from the inequality  $\|s^{(p)}(a^{(k)})\|_\infty \leq P(v_k \neq \infty)^{-1/\alpha}$ , it is obvious that all of the atomic martingales  $\{a^{(k)}\}_{k \in Z}$  belong to the space  ${}_p H_\alpha^s(B)$ .

Theorem 1 is proved.

**Theorem 2** Let  $1 < p < +\infty$ ,  $0 < \alpha \leq p$ ,  $B$  is a Banach space. If the stochastic basis  $\{F_n, n \in N^2\}$  is regular, then for every two-parameter  $B$ -valued martingale  $f = (f_n, n \in N^2) \in {}_p H_\alpha^s(B)$ , there exists a sequence of  $A^{s(p)}(\alpha, \infty)$  atomic martingales  $\{a^{(k)} = (a_n^{(k)}, n \in N^2)\}_{k \in Z}$  with the corresponding stopping time  $v_k$  and a sequence of positive numbers  $\{\mu_k\}_{k \in Z} \in l_\alpha$ , such that for all  $n \in N^2$ ,

$$f_n = \sum_{k \in Z} \mu_k a_n^{(k)}, \text{ a.e.,}$$

and

$$\|f\|_{p, H_\alpha^s(B)} \approx \inf \left( \sum_{k \in Z} \mu_k^\alpha \right)^{\frac{1}{\alpha}} < \infty,$$

where the infimum is taken over all possible decompositions of  $f$ . What's more, all of the atomic martingales  $\{a^{(k)} = (a_n^{(k)}, n \in N^2)\}_{k \in Z}$  belong to the space  ${}_p H_\alpha^s(B)$  too.

**Proof** Let  $F_k = \{S^{(p)}(f) > 2^k\}$  and

$$v_k = \inf\{n \in N^2 : E_n \chi(F_k) > \frac{1}{2R_p}\}, \quad (\inf\{\emptyset\} = \infty),$$

where  $R_p$  is the regular constant of  $\{F_n, n \in N^2\}$  such that  $\|d_n f\|^p \leq R_p E_{n-1} \|d_n f\|^p$ . Define

$$(U) = \sum_{n \in N^2} E(\chi(S^{(p)}(f) \leq 2^{k+1}) E_{n-1} (\|d_n f\|^p \cdot$$

$$\chi(v_k \ll n \not\gg v_{k+1}))) ,$$

$$(V) = \sum_{n \in N^2} E(\chi(S^{(p)}(f) > 2^{k+1}) E_{n-1} (\|d_n f\|^p \cdot$$

$$\chi(v_k \ll n \not\gg v_{k+1}))) .$$

Then we have

$$(V) \leq \sum_{n \in N^2} E(R_p \chi(S^{(p)}(f) > 2^{k+1}) E_{n-1} (\|d_n f\|^p \cdot$$

$$\chi(v_k \ll n \not\gg v_{k+1}))) = R_p \sum_{n \in N^2} E(E_{n-1} \chi(F_{k+1}) \cdot$$

$$E_{n-1} (\|d_n f\|^p \chi(v_k \ll n \not\gg v_{k+1}))) \leq$$

$$\frac{1}{2} \sum_{n \in N^2} E(\|d_n f\|^p \chi(v_k \ll n \not\gg v_{k+1}))) .$$

The rest of proof is similar to Theorem 1, for which we omit the details.

Theorem 2 is proved.

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## 两指标 $B$ 值鞅的原子鞅分解

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**摘要:** 通过引入两指标  $B$  值原子鞅, 讨论了由  $p$ -均方函数和条件  $p$ -均方函数定义的 Hardy 鞅空间  ${}_p H_\alpha^s(B)$  和  ${}_p H_\alpha^s(B)$  中两指标  $B$  值鞅的原子分解, 与原子函数不同, 所得原子分解结果不受 Banach 空间几何性质的限制, 推广了单指标  $B$  值鞅的相关结论.

**关键词:** 两指标  $B$  值鞅; 原子鞅; 原子分解

(责任编辑 韩 超)